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INTRODUCTION TO PHYSICAL OCEANOGRAPHY

UNIVERSITY OF VICTORIA

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Foreword

These are the course notes for Descriptive Physical Oceanography, taught at the University of Victoria, School of Earth and Ocean Sciences, a 13-week undergraduate course.

Unlike many physical oceanography courses, students taking this course are not expected to have studied vector calculus or differential equations, nor are they expected to have learned classical mechanics beyond what is typically offered in a first-year physics course. Many of the students have a deep interest in understanding the ocean and often approach the course with considerable intuition. The challenge of a course like this is to offer more than just a list of facts, but rather to try to give a feeling for the forces that drive the ocean, without becoming too bogged down in mathematical formalism. As such, this course tries to strike a balance between knowing about the various physical phenomena in the ocean and understanding the dynamics that give rise to them, while still giving students some practice in practical calculations.

The course is divided into three unequal parts: overturning circulations driven by lateral buoyancy differences and mixing; waves and tides; and wind-driven circulations. We start with estuaries as a nice introduction to overturning circulations, but also because the students do work in Saanich Inlet, and understanding estuaries early is very helpful.

Part I

Overturning circulations

1

Estuaries



Estuaries are semi-enclosed basins of water that are strongly influenced by freshwater sources (i.e. a river) or evaporation, yet have an opening to the open ocean where salty water can get into the basin. The semi-enclosed nature of the basin allows time for mixing between the fresh and the ocean water, and the density differences created drive a strong *overturning circulation* in the basin, such that much more water is drawn up the estuary than starts out flowing in from the river. Here we attempt to describe how this circulation works. How estuarine circulation occurs is subtle, but not terribly difficult to understand qualitatively. It is, however, not amenable to simple mathematical description, which may explain its omission from basic texts.

It is a little unusual to start a course with something as dynamic as an estuary. We do this as a bit of foreshadowing for the rest of the course, and because Saanich Inlet is an estuary, and we will be considering Saanich Inlet in the context of the course project.

1.1 Pressure differences

Before we start, it is helpful to think about how pressure differences drive flows. A fluid moves due to friction (i.e. the wind blowing over the surface), direct action (i.e. a paddle pushing water), or pressure forces. Pressure forces are unique to fluids, and are the forces that the water can exert on itself. In general they tend to move water so that pressure differences are evened out, i.e. the water pushes itself from high pressure regions to low pressure regions.

If the sea-surface is tilted, then the pressure will be greater where the sea surface is high than where it is low; in figure 1.1, $P_1 > P_2$. This will drive the flow from left to right. *Note* pressure increases with depth, so when we think about how the pressure moves water, we need to think about the pressure along a “geopotential” surface (ie. a surface along which the water would be flat). For our purposes, this is a set distance below the flat surface of the water.

In general, it is a good rule to think that the water will move in the direction that will lead to a flattening of the surface. The resting state of the body of water is to have a flat surface, as indicated with the dashed line at the surface in figure 1.1.

The situation is more complicated if we add density variations. Consider just tipping the interface of a two-layer fluid (figure 1.2), where the lighter fluid floats on top of the denser ($\rho_0 < \rho_1$; tilting this interface is actually hard to do, but just imagine). At first, the pressure difference in the upper layer is zero ($P_3 = P_4$) because the upper interface is not tilted. However, there is more dense water on the left side than the right, so there is a pressure gradient below the interface, $P_1 > P_2$, driving deep water to the right.

This leads to more water on the right hand side than the left side (figure 1.2b). This makes a surface pressure gradient that tends to drive the upper layer to the left ($P_4 > P_3$). This surface pressure gradient is set up *very* quickly, and only involves a tiny amount of water in order to drive the surface-layer flow to the left.

The water column will slosh around for a while, but friction will eventually damp the motions and the interfaces will be flattened as indicated with the dashed lines (figure 1.2c).

Again, the general idea is that the body of water has pressure gradients in it that drive a flow that will tend to flatten the layers.

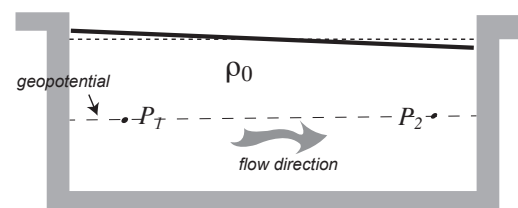


Figure 1.1: A body of water of uniform density with a tilted upper surface. Note that P_1 and P_2 are at the same geopotential height (i.e. their value of z is constant)

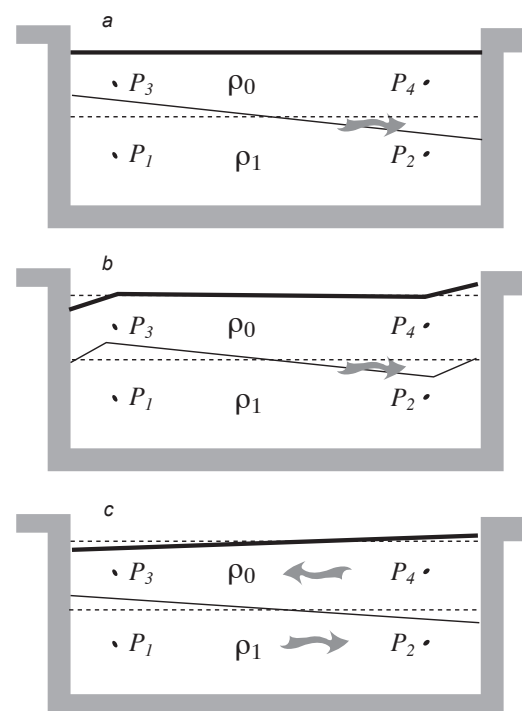
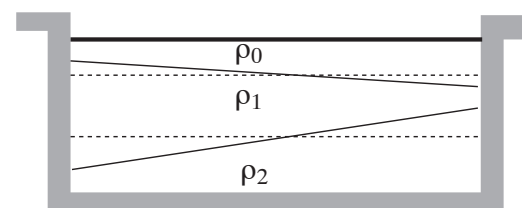


Figure 1.2: A two-layer body water with an interface that is initially tilted. See text for the description.



1.2 Estuarine Flow

1.2.1 Salt-wedge estuary

Estuaries are bodies of water with opposing buoyancy sources at their ends, usually a river at the head and the ocean at the mouth. The buoyancy difference usually arises from the salt content of the water, with fresh water being “lighter”, or more buoyant, than salty water. The simplest situation is a river spills into the ocean, spreads out and becomes a buoyant plume (figure 1.4). If we quantify the river volume transport as R (typically in units of $m^3 s^{-1}$) then in **steady state** the transport out the mouth of the estuary will be $Q_o = R$. The deep ocean layer will be stagnant.

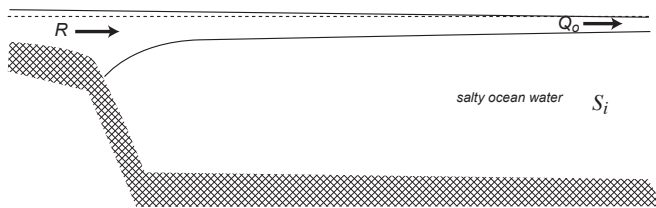


Figure 1.4: A salt-wedge estuary and river plume. The river discharges into the salt water without substantial mixing.

This type of estuary is called a *salt-wedge estuary*. The mouth of the Fraser River is a classic example of a salt-wedge estuary with a spreading plume (figure 1.5). Fresh water runs over salty with a small amount of mixing between the two layers. The salinity goes from 2 psu to 24 psu (1 psu $\approx$ 1 part-per-thousand, or gram of salt per kilogram of water) in just a few meters vertically. This sharp front moves back and forth with the tide, a typical feature of most salt-wedge estuaries.

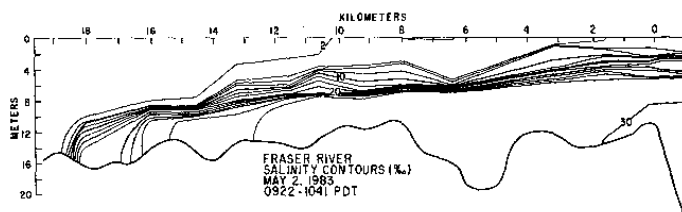


Figure 1.5: Salinity contours in mouth of the Fraser River. Salinity has units of parts per thousand, or ppt. So 1 ppt means 1 gram of salt per kilogram of water. The unit “psu” is also often used, meaning practical salinity unit, with 1 psu = 1 ppt. The river is on the left, the ocean on the right. [?]

1.2.2 Fjord-type estuary

Unlike a *salt-wedge estuary*, most estuaries have an “exchange flow” that is much stronger than the river flow. This flow is driven by vertical mixing that in turn creates tilted “isopycnals”, or constant density surfaces. These tilted isopycnals want to flatten out due to the pressure gradients that are set up, as discussed above.

As an example, imagine the situation pictured in figure 1.6. Here a river flows into salty water. If turbulent mixing is driven by strong tidal flow over the topographic bump in the middle of the estuary, then intermediate-salinity water will be formed. This intermediate-salinity water will be denser than the river water, but lighter than the ocean water, and will set up a flow that is pictured in figure 1.6b.

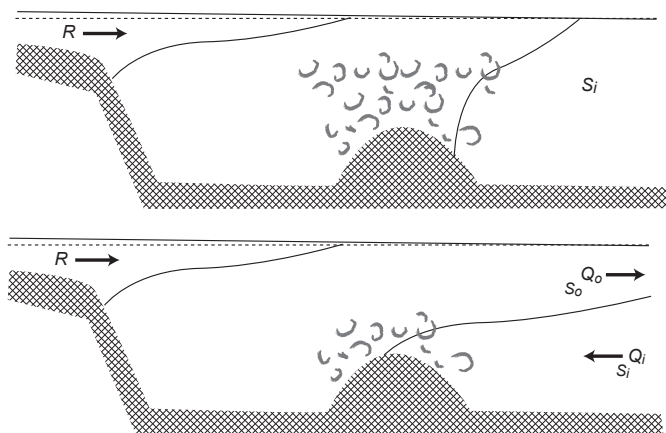


Figure 1.6: The effect of weak local mixing on flow in a fjord-type estuary. Intermediate salinity water is formed by the mixing which drives a flow out to sea. In this figure, “isohalines” are indicated by the contours (constant salinity surfaces) and will correspond to “isopycnals” (constant density surfaces) in most cases.

Determining the steady-state for such a flow is not trivial, and not very conducive to theoretical formulation. However, general tendencies can be determined. If there is more mixing, more intermediate water is formed. In steady state, this intermediate water must be flushed away, so there is a stronger circulation. Conversely, if the mixing is weaker, less intermediate water is produced, weakening the circulation.

1.2.3 Flood-plain estuaries

The more mixing that takes place, the more vertical the isopycnals, and the stronger the exchange flow relative to the river flow. The salt and density fields also become much more complicated. In shallow estuaries, of which the Chesapeake Bay is an excellent example, mixing from bottom stress and the winds tends to affect the whole water column, not just near topographic features. This sets up large along-estuary salinity and density gradients (figure 1.7). These types of estuaries tend to have much stronger amplification of the river flow (R) because of the extra mixing.

An example from an arm of the Chesapeake demonstrates how such an estuary may look (figure 1.8). Up estuary of Maryland Point, the estuary is really a river with no salinity. However, unlike the salt-wedge estuary, there is a gradual horizontal gradient of many kilometers before salty water is found. Note that even at this point the

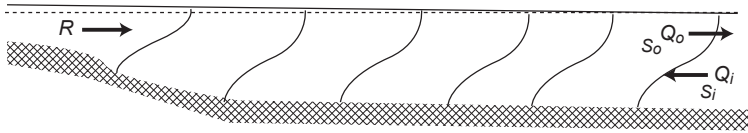


Figure 1.7: A flood-plain estuary schematic. Salinity increases from the head to the mouth, with weak vertical gradients.

water is still only 12 psu, and that the Chesapeake Estuary continues for hundreds more kilometers before open-ocean water is found.

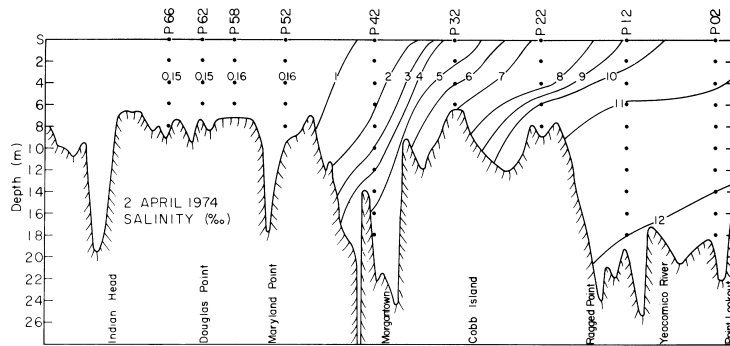


Fig. 8. Salinity section, April 1974.

Figure 1.8: A salinity cross-section from the Upper Potomac estuary, part of the Chesapeake Bay estuarine system [?].

The whole estuary has been measured and simulated numerous times. A recent example is shown in figure 1.9. Here the upper layer of fresher water can be seen flowing south and seaward. There is a strong return flow in the bottom 20 m. Note that the volume transport of the return flow is not accurately represented by these velocities, since they do not take into account the fact that the Bay widens considerably as it travels further south.

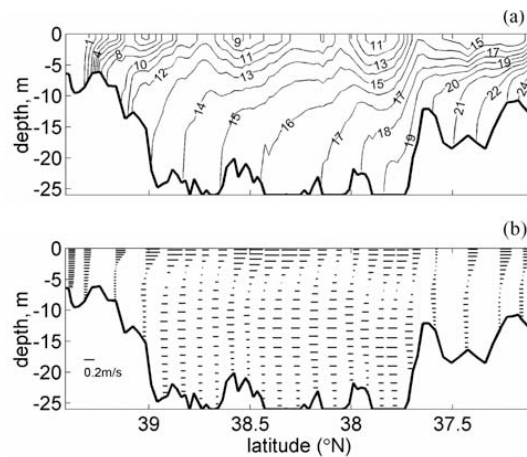


Figure 1.9: A salinity and mean velocity sections from a numerical model of the whole Chesapeake Bay [?]

Also note the lenses of fresher water that appear along the estuary. These occur because of rivers along the length of the estuary. This

complicates the simple picture given above, but the general trend of the flow remains unchanged.

1.2.4 *Time-dependence*

Time dependence changes the steady-state balance in estuaries in three main ways. First, the fresh-water input at the head of the estuary can change (figure 1.10). This significantly changes the salinity content of the estuary and the estuarine circulation. Somewhat paradoxically, the increased river flow can lead to a decreased estuarine flow. For instance in the Apr 1968 panel the isopycnals are much less tilted than in the Oct panel, indicating enhanced flow. The balance here, however, is not a perfect one, and determining the exact response if the river forcing changes depends on the mixing response. Strong vertical density gradients caused by fresh water influx can suppress turbulence and hence reduce the mixing required to drive the exchange flow.

A second effect that can change the forcing of an estuary is that the ocean water at the mouth of the estuary can change salinity due to seasonal changes in the open ocean. This tends to be a smaller effect because the salinity differences, even in coastal waters, tend to be less pronounced. However, coastal waters are subject to seasonal changes due to coastal upwelling and downwelling. During upwelling, denser water can be found at the estuary mouth, whereas during downwelling the density can decrease.

Finally, there is a time dependence to the mixing that ultimately drives the enhanced estuarine circulation. The most regular source of time-dependence is the tidal forcing. This varies with the fortnightly (every two weeks) modulation of the tide, and with the long-term perigee/apogee variations of the moon-earth-sun system. More mixing during the *spring* tides leads to *isohalines* that are more tilted (figure 1.11) which therefore drive more circulation.

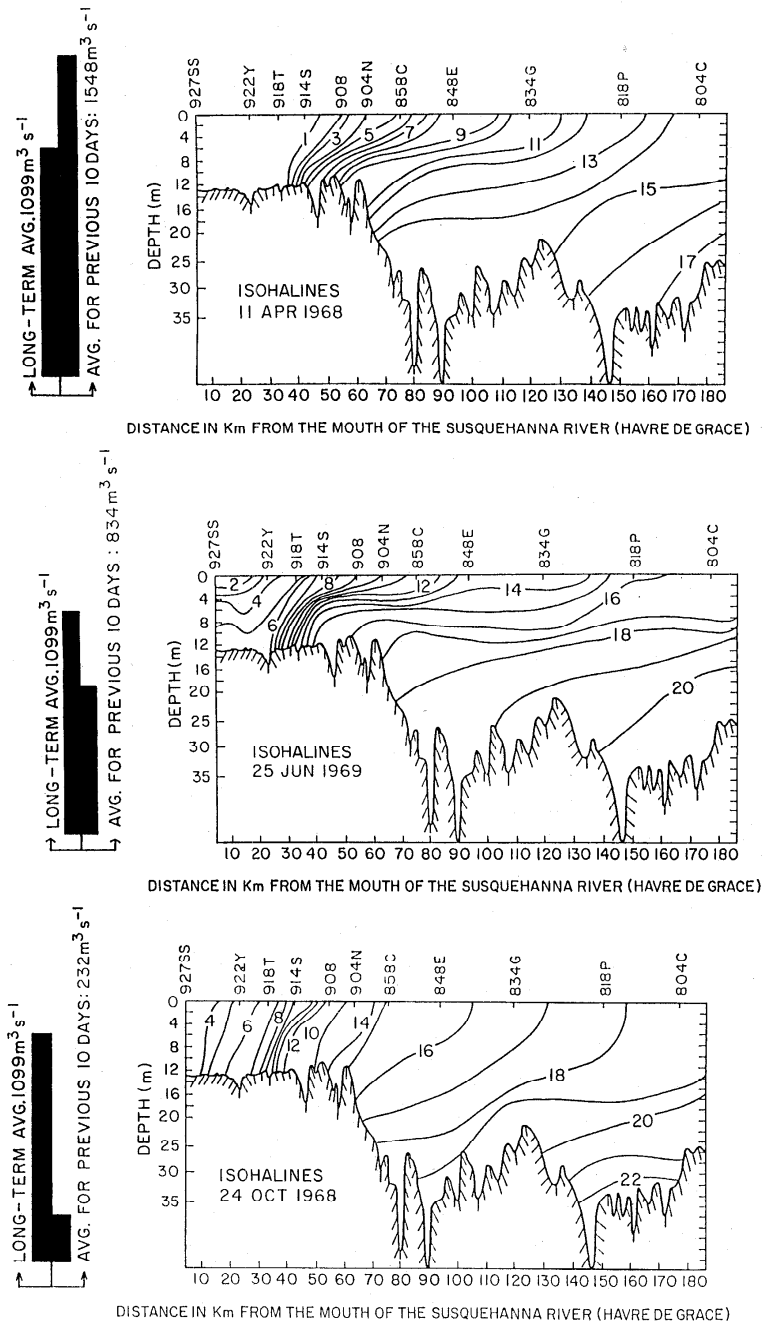


Figure 1.10: Salinity in the Chesapeake as it varies due to river discharge over the year (bars on left) [?].

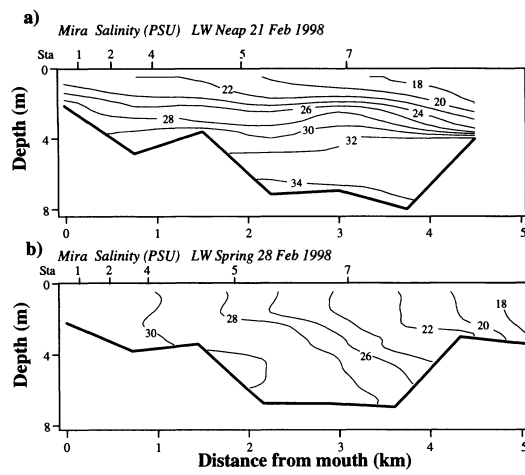


Figure 1.11: Salinity in the Mira estuary (Portugal) during neap and spring tides [?].

1.2.5 Topographic blocking (mostly in fjords)

One final topic is topographic blocking. Fjords are often demarcated from the ocean by underwater sills (terminal moraines left over from glaciations). These sills can be quite tall and sometimes the tides are not strong enough to bring the densest water from the ocean-side of the sill over the sill (figure 1.12a). The estuarine circulation carries on as normal above this depth, but the deep ocean layer does not make it into the landward basin. This is called “blocking”.

However, occasionally strong tides perhaps enhanced by atmospheric forcing will conspire to provide enough energy for the densest water to get over the sill. This dense water will spill over the sill and, if it does not mix, will reach the bottom of the fjord (figure 1.12b). For some fjords this happens every spring-neap cycle. For others only when the spring tide is particularly large, or the water on the seaward side denser than usual.

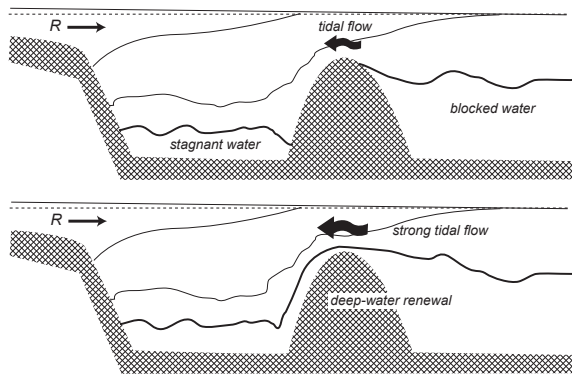


Figure 1.12: A narrow-silled fjord with blocking of deep water, and a deep-water renewal event.

Conversely, if the sill is broad, or has a lot of horizontal constrictions, deep-water renewal can happen on the exact opposite phase of the spring-neap cycle (figure 1.13). During spring tides the incoming dense water is subjected to so much mixing that it is mixed away before it can reach the inner basin. It is only during neap tides with enough velocity to push the dense water into the inner basin, but not so much as to cause excessive mixing that deep-water renewal can take place.

1.3 Quantifying the Circulation: Knudsen Relation

Estuaries are a clean example of conservation of volume and salt. In steady state, whatever flows in must flow out — both water and salt. Writing that down carefully lets us estimate exchange flows that are otherwise hard to measure directly.

The key insight is that the outflowing surface water has some

See also Open University Sec. 6.2

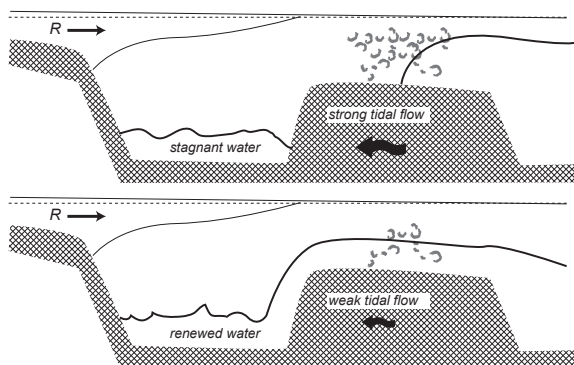


Figure 1.13: A broad-silled fjord with blocking of deep water during spring tides, and a deep-water renewal during neaps.

salinity (S_o in figure 1.7), even though the river delivers fresh water. That means the river water has picked up salt by mixing with the water below. It also means the outflow must be larger than the river flow ($|Q_o| > R$) and that there must be an inflow from the ocean (Q_i) to replace the salty water that has been mixed upward. Salinities S_o and S_i are relatively easy to measure from a ship; the volume transports Q_o and Q_i are not. Conservation of volume and salt gives us two equations in two unknowns, which we can solve given R from river gauge data.

1.3.1 The Conservation of Volume

Suppose we are interested in the dynamics of a body of water, say a rectangular swimming pool. The pool has a hose pumping water into it at a volume transport of q [$\text{m}^3 \text{s}^{-1}$]. If its surface area is A , how fast is the water rising in the pool? The answer is $w = q/A$: the volume transport divided by the surface area.

More generally, in steady state the total volume flowing in must equal the total volume flowing out:

$$\sum_i q_i = 0 \quad (1.1)$$

where transports into the volume are negative and transports out are positive (or vice versa, as long as you are consistent, but we will try to use this convention since it is the one used in the mathematics literature).

1.3.2 Conservation of salt

Salt is carried into or out of a volume by the flow. The rate of salt transport is simply the volume transport multiplied by the salinity:

$$q_s = S q \quad (1.2)$$

with units of $\text{psu} \cdot \text{m}^3 \text{s}^{-1}$. In steady state the salt transported in must equal the salt transported out. We will use this, along with conservation of volume, to estimate estuarine exchange flows below.

Box 1.3.1: Examples of conservation of mass

Q: Suppose we have a vat, with volume $V = 10 \text{ m}^3$ of water in it. A hose flows into it at a rate of $0.1 \text{ m}^3/\text{s}$ and a second hose flows out at the same rate. If the concentration of Caffeine in the vat is initially $0 \text{ g}/\text{m}^3$, and $10 \text{ g}/\text{m}^3$ in the hose, what is the rate that caffeine is being added to the tank?

A: it is simply $qC = 1 \text{ g}/\text{s}$.

Q: What rate is caffeine leaving the tank initially?

A: The initial concentration in the tank is zero, so $qC = 0 \text{ g}/\text{s}$.

Q: Assuming the tank is well mixed, what is the rate of change of the concentration in the vat, initially?

A: This is just the rate of change of the amount of caffeine, $1 \text{ g}/\text{s}$ divided by the volume of water in the vat, so $0.1 \text{ gm}^{-3} \text{s}^{-1}$.

Q: New vat, with three hoses. One hose has $C_1 = 10 \text{ g}/\text{m}^3$ of caffeine and flows at $q_1 = 0.1 \text{ m}^3/\text{s}$. The second hose flows in with $C_2 = 5 \text{ g}/\text{m}^3$, and $q_2 = 0.4 \text{ m}^3/\text{s}$. If the vat is well-mixed and in steady state, what is the flow out the third hose, and what is the concentration of the caffeine in the water in the vat?

A: First, in steady state, the volume fluxes equal zero (there can be no net flow into the vat), so $q_1 + q_2 + q_3 = 0$. We know q_1 and q_2 , so we know $q_3 = -0.5 \text{ m}^3/\text{s}$, where the negative sign means water is flowing out.

We also know that there can be no net flow of caffeine, so $C_1 q_1 + C_2 q_2 + C_3 q_3 = 0$. We know everything except for C_3 , so we solve and get $C_3 = \frac{C_1 q_1 + C_2 q_2}{-q_3} = 6 \text{ g}/\text{m}^3$.

1.3.3 Application to Estuaries: The Knudsen Relation

Imagine the flow at the estuary mouth as two layers: salty ocean water flowing in along the bottom with volume transport Q_i and salinity S_i , and fresher water flowing out at the surface with volume transport Q_o and salinity S_o . The river delivers fresh water at the head with volume transport R .

We adopt the convention that *positive transport is out of the estuary*. So $Q_o > 0$ (water leaving), while $Q_i < 0$ and $R < 0$ (water entering). This feels odd for the river, but it keeps the math consistent: the

steady-state volume budget is simply

$$Q_o + Q_i + R = 0 \quad (1.3)$$

with no need to worry about which terms go on which side.

Q: What is the salinity of the river water?

Q: Write down the steady-state salt budget. What is the salt transport (in $\text{psu} \cdot \text{m}^3 \text{s}^{-1}$) carried by each of the three flows? Note that the sign of each salt transport follows from the sign of the volume transport.

Q: You now have two equations and two unknowns (Q_i and Q_o), given measured values of R , S_i , and S_o . Solve for Q_i and confirm that:

$$Q_i = \frac{S_o R}{S_i - S_o}. \quad (1.4)$$

Check: with $R < 0$ and $S_i > S_o$, does Q_i have the right sign?

Q: If there is no mixing, what is S_o , and how big is Q_i ?

Q: If there is a lot of mixing, what happens to $S_i - S_o$ and thus Q_i ?

These results are powerful because Q_i and Q_o are hard to measure directly, while R , S_i , and S_o can be estimated from river gauge data and a few CTD casts.

The traditional form of the Knudsen relation defines all transports as positive magnitudes and writes the budgets with explicit signs: $Q_o = Q_i + R$ and $S_o Q_o = S_i Q_i$. The result is the same equation for Q_i , but with $R > 0$. Our sign convention is more consistent with the general volume-budget framework used later in the course.

1.4 Exercise

We will practice some of what we have learnt based on a paper about local waters by ?. As part of your final project you will read about similar processes in Saanich Inlet [?].

Q: Consider figure 1.14, which shows the salinity observed and modeled in the Strait of Juan de Fuca. Based on these plots, where do you think the mixing is the strongest in the Strait?

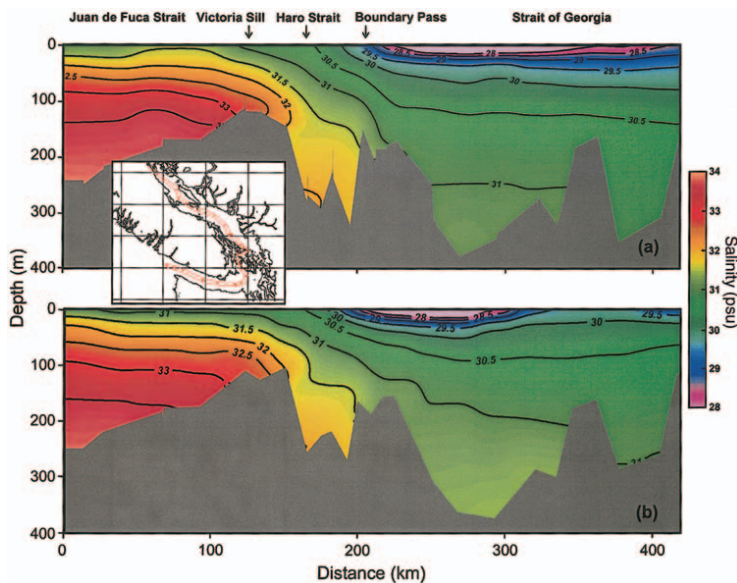


Figure 6. Along-strait section of annual mean salinity from observations (a) and the reference experiment (b).

Figure 1.14: Observed and modeled salinity in the Straits of Juan de Fuca and Georgia [?]. The ocean is on the left, and the Fraser River is at Distance = 270 km. The upper plot (a) are the observations and the lower (b) a numerical simulation.

Q: For the flow in figure 1.14, sketch where you think the water is flowing. What happens north of the Fraser river (at Distance = 250 km; make sure you know where north is on this plot)?

Q: Consider figure 1.15, which compares two numerical model runs, one with tides and one without. Which is which, and why? Which has the stronger horizontal circulation?

Q: A seasonal time series of the salinity in Haro Strait is given in figure 1.16. What features of the flow can you identify that indicate the time dependence of the estuarine flow? Pay particular attention to the modeled timeseries (which has better temporal resolution)?

Q: Use the Knudsen relation on the two data plots in figure 1.15 to estimate the exchange flow if the river input is $10^4 \text{ m}^3 \text{ s}^{-1}$. Which case has a stronger exchange, a) or b)?

Q: Suppose the river is suddenly damned, so $R = 0 \text{ m}^3 \text{ s}^{-1}$. The estuary's average salinity will (initially) change at what rate (in units of psu s^{-1})? Hints: first, you should assume that the estuary adjusts so the flux in is equal to the flux out the mouth. Second, you need to

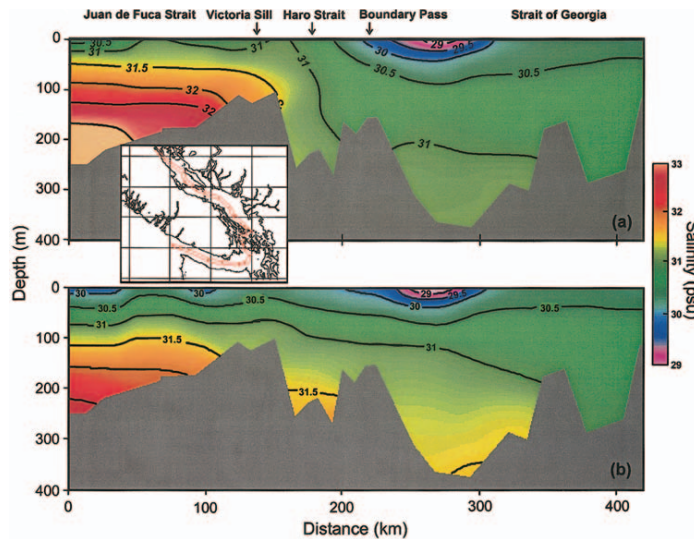


Figure 1.15: Salinity for two model runs, one with and the other without tides [?].

estimate the volume of the estuary to answer this questions. Rough numbers are fine, based on the figures above.

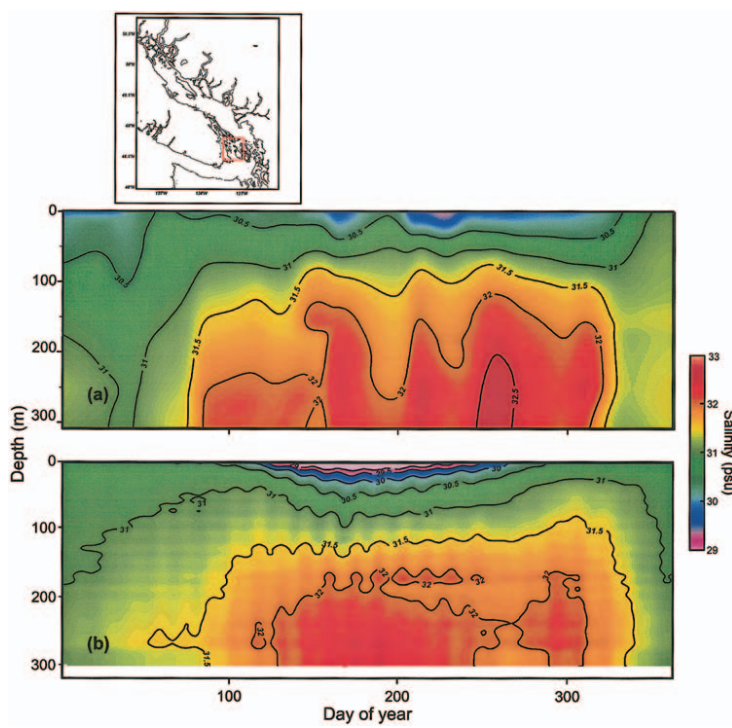


Figure 1.16: Time evolution of observed and modeled salinity in Haro Strait [?].

Box 1.4.1: Practice questions (not the exercise!)

Some students are not comfortable with volume and mass balances. If that is the case, try some extra practice questions below. Please ask me if you need more help with these!

- Suppose the water level of a straight-sided pool is observed to rise at 0.01 m/s , and has a surface area of 50 m^2 . There is one hose filling it with speed 4 m/s and cross-sectional area 0.01 m^2 , and a second hose filling it with a cross sectional area of 0.02 m^2 . What is the flow speed in the second hose?
- Suppose a pipe has a diameter of 0.1 m and a flow speed of 1 m/s . If the pipe narrows to 0.05 m what is the flow speed at that part of the pipe (in steady state)?
- Suppose there is a vat with 200 g/m^3 solution of chlorine being fed into the pool through a hose with diameter 0.1 m at a rate fast enough to replace evaporative losses in the pool of $0.01 \text{ m}^3/\text{s}$. What is the transport of chlorine into the pool?
- If the average concentration of chlorine in the pool is initially 50 g/m^3 and 1000 s later is measured to be 55 g/m^3 , what is the volume of the pool?

1.5 Example problems

These problems give more practice with the ideas above.

1. A river delivers freshwater to an estuary at $R = -500 \text{ m}^3 \text{ s}^{-1}$ (recall our sign convention: transport into the estuary is negative). At the mouth, the outflowing surface layer has salinity $S_o = 15 \text{ psu}$ and the inflowing bottom layer has salinity $S_i = 32 \text{ psu}$.
 - Write down the volume and salt budgets for the estuary.
 - Solve for the inflow Q_i and outflow Q_o (in $\text{m}^3 \text{ s}^{-1}$).
 - How many times larger is the outflow than the river flow itself? This ratio is a rough measure of how much mixing is going on in the estuary.
2. Two estuaries are fed by the same river, $R = -1000 \text{ m}^3 \text{ s}^{-1}$, but have different exchange flows: estuary A has $S_o = 5 \text{ psu}$ and $S_i = 30 \text{ psu}$ (strongly stratified, salt-wedge-like), while estuary B has $S_o = 20 \text{ psu}$ and $S_i = 30 \text{ psu}$ (well mixed). Use the Knudsen relation to find Q_i for each estuary, and say which one has the

stronger exchange flow. Does this match your intuition about mixing and exchange flow from the text?

3. A fjord has a volume of $V = 5 \times 10^8 \text{ m}^3$. Over a survey, the inflow is measured at $Q_i = 2000 \text{ m}^3 \text{ s}^{-1}$ with $S_i = 31 \text{ psu}$, and the outflow at $Q_o = 2200 \text{ m}^3 \text{ s}^{-1}$ with $S_o = 28.5 \text{ psu}$.
 - Is this fjord in a salt-conservation steady state? If not, is its average salinity increasing or decreasing, and at what rate (in psu s^{-1})?
 - Roughly how long would it take to change the average salinity by 1 psu at this rate?
4. A vat used to model an estuary has three hoses: hose 1 delivers fresh water ($S_1 = 0 \text{ psu}$) at $q_1 = 0.05 \text{ m}^3 \text{ s}^{-1}$, hose 2 delivers salty water ($S_2 = 35 \text{ psu}$) at $q_2 = 0.02 \text{ m}^3 \text{ s}^{-1}$, and hose 3 is the only outflow. The vat is well-mixed and in steady state.
 - What is the flow q_3 out of hose 3?
 - What is the salinity of the water leaving through hose 3?
 - Which inflow hose is playing the role of the river, and which is playing the role of the ocean inflow in a real estuary? What is missing from this vat model that a real estuary has (hint: think about how many layers the outflow needs)?
5. Consider a fjord with a long, shallow sill. Sketch the exchange flow through the sill during spring tide and during neap tide, being sure to indicate the river and ocean sides of the fjord.
 - Indicate the flow directions at each tide.
 - Which tide has the stronger net (tide-averaged) exchange flow?
 - Which tide is more likely to produce a deep-water renewal event, and why?
6. Two estuaries have the same geometry, river input, and ocean forcing, but one has relatively weak mixing and the other very strong mixing.
 - Sketch what the isohalines might look like in each estuary (make the difference clear, and make sure the isohalines you draw are physically realizable).
 - Sketch the exchange flow in each estuary, and indicate which one has the stronger exchange flow.

Part II

Waves and Tides

Part III

Wind-driven circulations