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INTRODUCTION TO PHYSICAL OCEANOGRAPHY

UNIVERSITY OF VICTORIA

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Foreword

These are the course notes for Descriptive Physical Oceanography, taught at the University of Victoria, School of Earth and Ocean Sciences, a 13-week undergraduate course.

Unlike many physical oceanography courses, students taking this course are not expected to have studied vector calculus or differential equations, nor are they expected to have learned classical mechanics beyond what is typically offered in a first-year physics course. Many of the students have a deep interest in understanding the ocean and often approach the course with considerable intuition. The challenge of a course like this is to offer more than just a list of facts, but rather to try to give a feeling for the forces that drive the ocean, without becoming too bogged down in mathematical formalism. As such, this course tries to strike a balance between knowing about the various physical phenomena in the ocean and understanding the dynamics that give rise to them, while still giving students some practice in practical calculations.

The course is divided into three unequal parts: overturning circulations driven by lateral buoyancy differences and mixing; waves and tides; and wind-driven circulations. We start with estuaries as a nice introduction to overturning circulations, but also because the students do work in Saanich Inlet, and understanding estuaries early is very helpful.

Part I

Overturning circulations

3

Heat and Salt fluxes

The ocean is moved by three main forcings - friction from the winds, body forces due to astronomical forcing i.e. the tides, and differences in buoyancy forcing. This section discusses the buoyancy forcing. The buoyancy of water is how dense it is, and as we saw in the previous chapter, it is determined by the temperature of the water and how much salt it has in it (and of course the pressure).

This is crucially important for the circulation of the ocean. We already saw how estuaries have differential buoyancy forcing due to a freshwater source at the head of the estuary being more buoyant than the salty ocean water. Buoyancy forcing by fresh water is also important on a global scale. Perhaps even more important is the differential heating, with the ocean gaining heat near the equator and losing it near the poles. The ocean carries roughly 30% of the total poleward heat transport; the atmosphere carries the rest. However, it also drives the overturning circulation of the global ocean.

Like an estuary, if the ocean only had heating and cooling at the surface the total volume transport of water in the overturning circulation would be weak. However, mixing brings heat to great depths in the ocean and this creates pressure gradients that move a large slow overturning.

3.1 Surface heat fluxes

Most heat exchange in the ocean occurs at the surface, with short wave radiation passing through the atmosphere and impacting the ocean, and then the ocean losing heat to back radiation, latent heat flux, and heating and cooling from direct contact with the atmosphere.

An estimate of the net heat flux in the ocean is given in figure 3.1 and spatially in figure 3.2e. Recall that heat is specified in units of Joules, where 1 Joule is the energy (or heat) required to warm a kilogram of water one degree Kelvin. Heat fluxes are a rate of heating,

and hence are in units of Watts, or Joules per second. If we want to make a map like figure 3.2e, then we want to know the rate of heating per area, and we plot in units of Watts per meter squared, or W m^{-2} . The average over the surface area of the ocean is shown in figure 3.1.

Overall, there is a net input from shortwave radiation of about $H_{SW} = 170 \text{ W m}^{-2}$ once sunlight penetrates the cloud cover (figure 3.1). There is a net longwave radiation of $H_{LW} = 53 \text{ W m}^{-2}$ just due to the temperature of the water and the temperature of the atmosphere and clouds above. The remaining two terms are “sensible” heat loss, which is just due to the temperature difference diffusing across the sea surface ($H_{sensible} = 16 \text{ W m}^{-2}$), and the “latent” heat loss due to evaporation, which is the largest loss ($H_{latent} = 100 \text{ W m}^{-2}$).

Note that the estimate in figure 3.1 has a net warming of 0.8 W m^{-2} , which is due to global warming. We can consider the average heating for a $D = 4000\text{-m}$ deep ocean using this number. The heat capacity of water is $c_p = 4.2 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$, and the density 1030 kg m^{-3} , so the rate of temperature change is $dT/dt = H_{net}/(D\rho c_p) = 0.0015 \text{ K/y}$, or greater than 600 years to increase by 1 degree at the current rate. This of course does not mean climate change is not important to the ocean - much of this change is confined to the upper few hundred meters, and climate change strongly affects winds that drive much of the ocean circulation.

3.1.1 Incoming shortwave radiation

The spatial distribution of these fluxes and their net magnitudes indicate that, unsurprisingly, the net incoming shortwave radiation is strongest at the equator (figure 3.2a). This radiation comes from the sun, and is modified as it passes through the atmosphere. Of course it includes all wavelengths of radiation, but it is very much dominated by short wavelength radiation, so it is called “shortwave”.

Estimating the radiation coming from the sun and impacting the ocean would be a simple matter depending only on the tilt of the earth, and to a (much) lesser extent whether the sun is in aphelion or perihelion. However, the atmosphere severely modifies the net flux reaching the earth, due to ozone and clouds. Indeed, much of the east-west spatial variability to the shortwave surface fluxes is due to clouds. For instance, the inter-tropical convergence zone can be clearly seen just north of the equator (figure 3.2a) as a region of reduced incoming radiation.

Given this, it is important to realize that it is very difficult to make accurate estimates of the incoming radiation over the full expanse of

the ocean. Satellites make this easier, because they give a good idea of cloud cover, but the thickness of the clouds and their humidity all matter to making an estimate. Radiation can be measured with **radiometers**, but these need ships, moorings, or other platforms to make direct measurements. Hence estimates of shortwave radiation are derived from empirical models.

Some idea of the magnitude of the problem can be seen by considering the net heat flux of 1.2 PW (figure 3.2e) integrated over the entire global ocean (1 PW is 10^{15} Watts). The volume of the oceans is approximately $1.335 \times 10^{18} \text{ m}^3$. The heat capacity of water is $4180 \text{ J kg}^{-1} \text{ K}^{-1}$ and its density approximately 1030 kg m^{-3} . Using these numbers, this 1.2 PW discrepancy in the annual mean would lead to a one-degree warming of the ocean over 140 years. This would be quite large and rapid, even on the scale of anthropogenic climate change, and this net should be considered a bound on how accurate the various terms of the budget are, rather than a direct estimate of how fast the ocean is warming.

We can see the effect of the earth's tilt and changing cloud cover if we look at seasonal averages of the shortwave radiation (figure 3.3a, figure 3.4a, figure 3.5a, figure 3.6a). Not surprisingly, during the boreal winter, the net radiation is low in the northern hemisphere compared to the boreal summer.

Note this is the same as the 2.3 PW discussed above, but derived a different way, and both methods have large error bars.

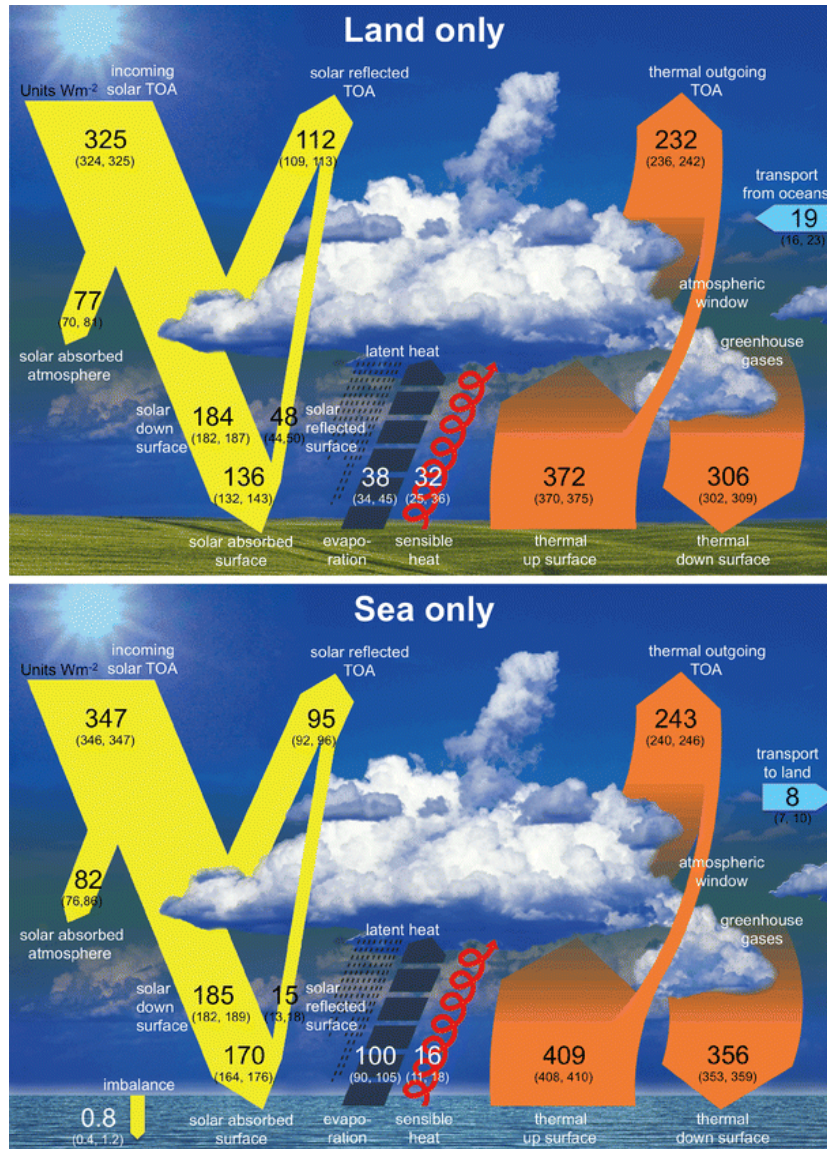


Figure 3.1: Heat fluxes over land and ocean, units of W m^{-2} . Note the net “thermal”, or longwave back radiation is $H_{LW} = 53 \text{ W m}^{-2}$ [?]. “Solar absorbed” is the same as H_{SW} below, and “evaporation” is the same as H_{latent} . Note the error bounds next to each estimate. The estimated net imbalance is the warming of the oceans due to global warming.

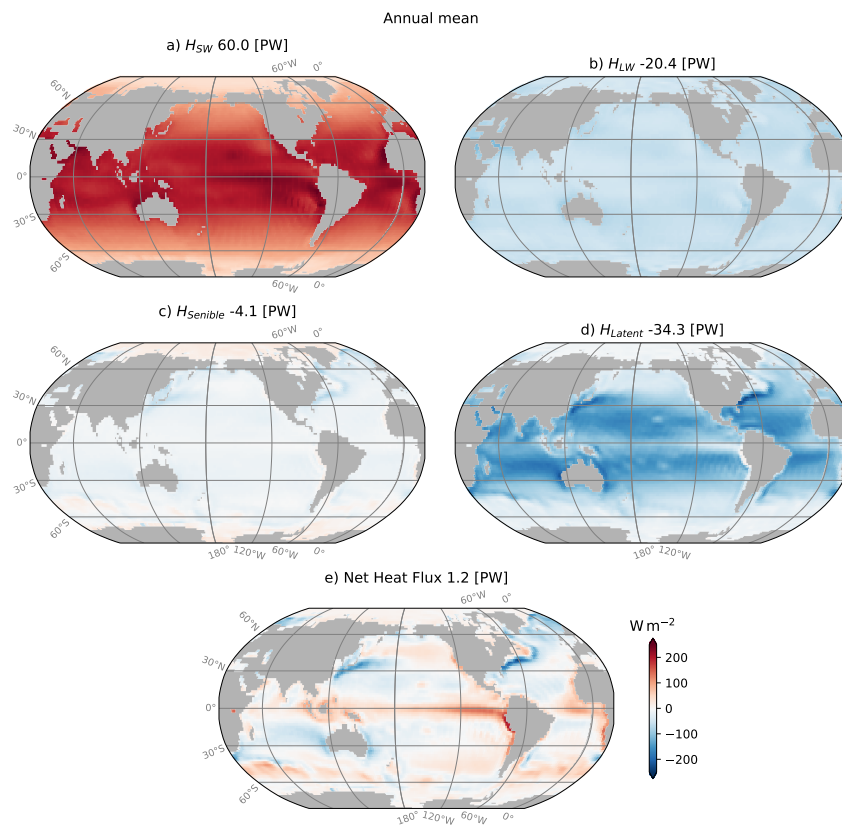


Figure 3.2: Net surface heat fluxes, annually averaged. a) Net shortwave heat flux, b) net longwave flux, c) sensible heat flux, d) latent heat flux and e) net heat flux. Data source: [NCEP1 reanalysis](#).

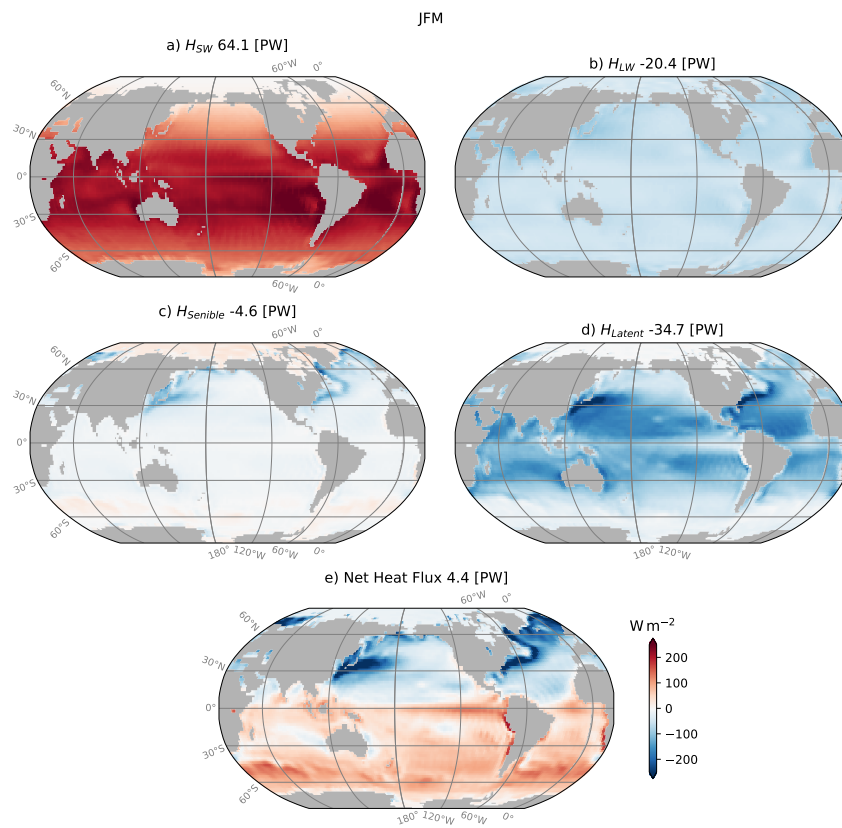


Figure 3.3: Net surface heat fluxes, January to March. a) Net shortwave heat flux, b) net longwave flux, c) sensible heat flux, d) latent heat flux and e) net heat flux. Data source: [NCEP1 reanalysis](#).

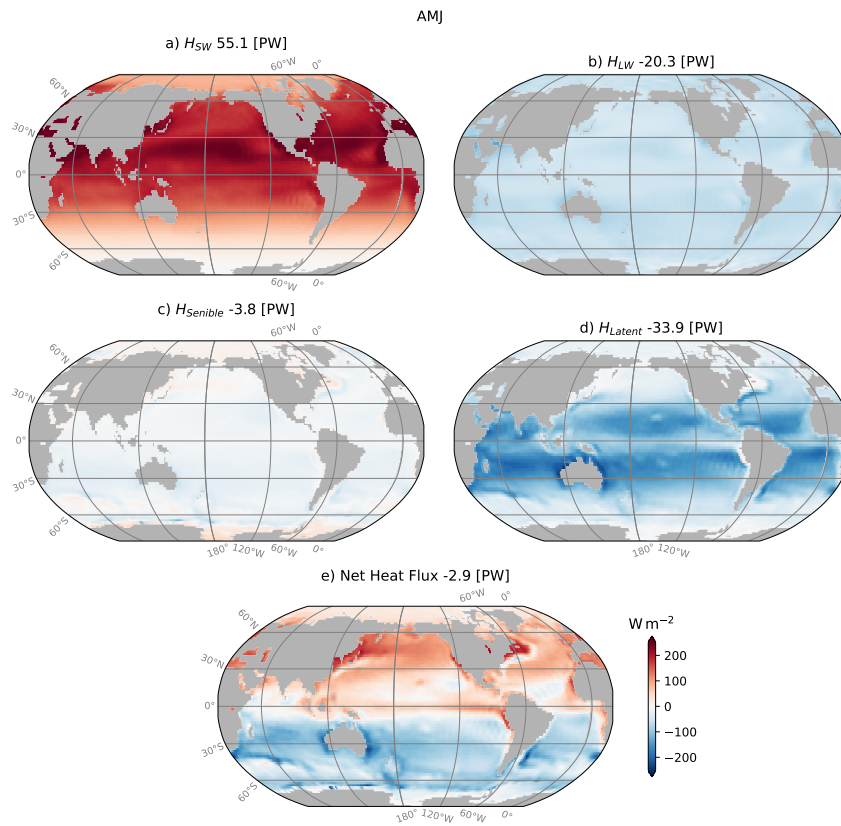


Figure 3.4: Net surface heat fluxes, April to June. a) Net shortwave heat flux, b) net longwave flux, c) sensible heat flux, d) latent heat flux and e) net heat flux. Data source: [NCEP1 reanalysis](#).

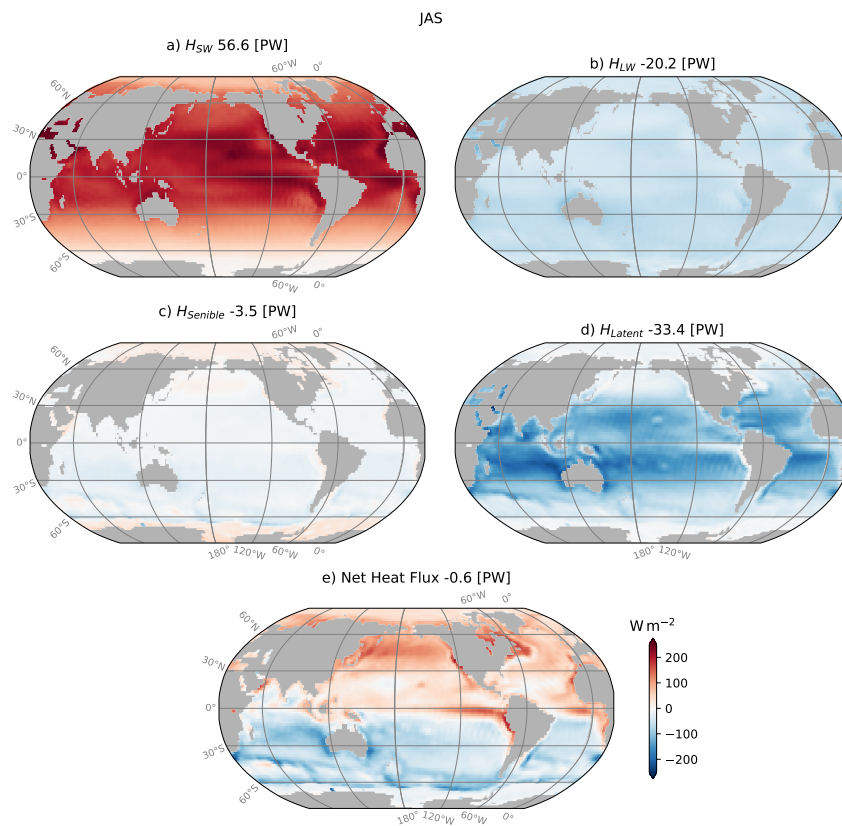


Figure 3.5: Net surface heat fluxes, July to September. a) Net shortwave heat flux, b) net longwave flux, c) sensible heat flux, d) latent heat flux and e) net heat flux. Data source: [NCEP1 reanalysis](#).

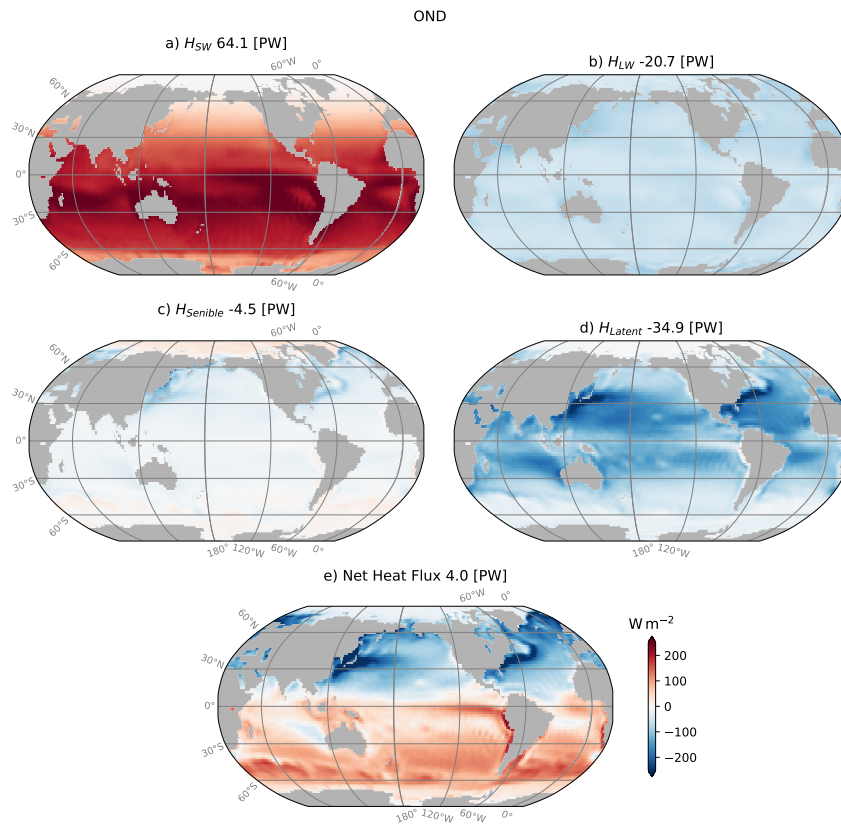


Figure 3.6: Net surface heat fluxes, October to December. a) Net shortwave heat flux, b) net longwave flux, c) sensible heat flux, d) latent heat flux and e) net heat flux. Data source: [NCEP1 reanalysis](#).

3.1.2 Longwave radiation (back radiation)

The earth and ocean emit radiation back into the atmosphere due to their temperature due to blackbody radiation. If there were no atmosphere, then H_{LW} would depend on temperature of the water to the fourth power, the Stefan-Boltzman constant, and the emissivity of the water. If this were the case, the ocean would have very large spatial differences in H_{LW} that depended the sea surface temperature. That is not the case (figure 3.2b) because the atmosphere acts as a strong insulator, and back-reflects much of the emission. Hence, the *net* longwave radiation is surprisingly uniform. This is true spatially, as well as temporally (figure 3.3b, figure 3.4b, figure 3.5b, figure 3.6b). It is indeed surprising that the net back radiation is strongest in the winter when the sea surface is usually colder.

This again points to the difficulty in estimating this flux. As with shortwave radiation it requires radiometers to measure directly, and some correlation with atmospheric forcing must be made to come up with a net value.

3.1.3 Sensible heat transfer

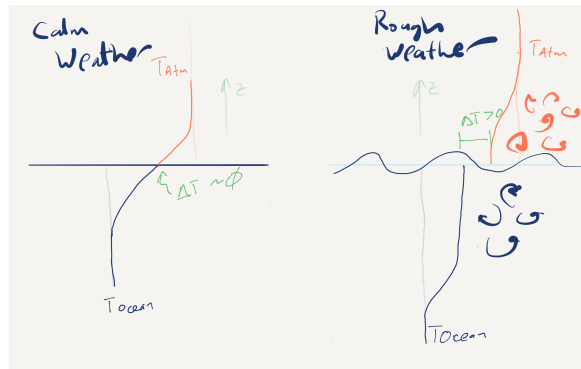


Figure 3.7: Sketch of ocean and atmospheric temperature profiles under calm weather (left) and rough weather (right). Under calm, stable conditions the temperature difference is confined to a thin layer at the interface and sensible heat transfer is weak. Under rough conditions, wave breaking and turbulence mix the interface layer, spreading the temperature difference over a thicker layer and greatly enhancing the transfer.

Sensible heat transfer is the weakest term, despite the fact it is probably what most people think of as how heat is transferred, i.e. when the atmosphere is cold the ocean loses heat and when it is warm it gains heat. The heat exchange is usually assumed to be driven by the temperature difference between the top of the ocean and the atmosphere, i.e. $H_{Sensible} \approx k(T_{air} - T_{ocean})$ where k is a transfer co-efficient ($W K^{-1}$). However, the temperature of the upper ocean and lower atmosphere of course depends on the flux. So if there is only weak mixing of heat by upper ocean turbulence, the temperature profiles will rapidly equilibrate in thin layers at the interface (figure 3.7a). Conversely, if there is substantial mixing, the temperature difference between the two layers can remain large

because heat is mixed away from the interface.

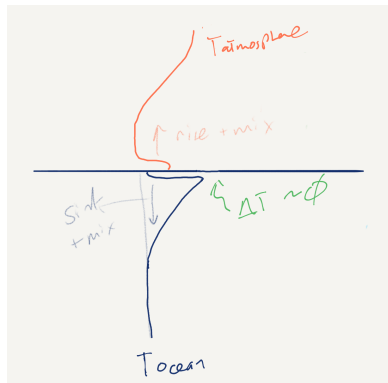


Figure 3.8: Sketch of an unstable interface. Cooling at the surface leaves cold water on top of warm, which sinks and mixes below the interface; the corresponding warming of the air above drives rising motion and mixing in the atmosphere. This convective mixing on both sides of the interface maintains the temperature difference that drives sensible heat transfer.

If the interfaces are unstable, then the ocean (and atmosphere) will tend to “convect”. i.e. if the atmosphere at sea level is colder than the ocean, then the cooling of the ocean will lead to cold water on top of warm water and that water will sink. As it does so, it mixes with the warm water below, and the mixed layer will tend to deepen. Again, mechanical mixing by the wind can enhance this process.

Needless to say, parameterizing these effects to come up with maps like figure 3.2c, has required many years of effort and specialized instrumentation figure 3.9. Measured heat fluxes, as we will see below, are notoriously hard to collect, as are the forcing parameters for those heat fluxes, involving wind speed, ocean current speed, and parameterization for the strength of turbulence in the lower atmosphere and ocean. For an overview of the state of the art, and the pressing need to improve these measurements see ?.

The sensible heat losses show a relatively straight forward patterns with the seasons, with a bit of warming of the ocean in the summer, and substantial cooling in the winter (figure 3.3c to figure 3.6c) . Particularly for the winters there is substantial east-west asymmetry in the ocean’s basins, for instance in figure 3.3c, there is strong losses off Japan and the Eastern US, centred at around 40 N. This is due to dry cold winds off the continents, blowing from west to east with the Jet Stream coming into contact with warm water being advected north by the Kuroshio and Gulf Stream.

3.1.4 Latent heat losses

Latent heat losses are those due to heat used to evaporate water. Evaporation occurs when the atmospheric humidity, q , is less than 100%. During evaporation water mass is transferred from the ocean to the atmosphere, and the heat stored in that water is transferred from the ocean to the atmosphere. This transfer is not 100% efficient



Figure 3.9: Platforms used to measure heat fluxes in the ocean, from ?.

because some energy is used to achieve the evaporation, but it does result in a substantial transfer of heat.

The strength of this transfer depends on the humidity of the ocean directly at the air-sea interface, and hence, just like for the sensible heat transfer, it depends on the turbulence in the atmospheric “boundary layer” - if the layer is turbulent because there are strong winds and/or convection, then the moist near-interface air is mixed more thoroughly and the evaporation and exchange of heat is enhanced. Note that indirectly the exchange depends on the temperature difference between the ocean and the air as cool air has relatively little moisture, and warm water is more readily evaporated.

Again, this leads to seasonal changes in the heat flux that mean more evaporative heat flux occurs in the winter than the summer, (figure 3.3d to figure 3.6d). Further there are clear east-west asymmetries across the basins with the western boundaries at mid-latitude being conspicuous for their evaporative heat loss. This is because cold dry continental air blows west-east across the warm western boundary currents at these latitudes, leading to substantial evaporation.

3.1.5 Integrated Surface Heat Fluxes

The maps in figure 3.2 to figure 3.6 can be integrated to get the net heat flux over the year (figure 3.10). Remember that there is a geometric effect of the area of the earth per degree of latitude at the pole goes to zero, whereas it is very large at the equator. Keeping this in mind, the integral around the globe for each degree shows that there is a net heat input at the equator, and losses to the north and south. Interestingly, in the southern hemisphere, there is a net heat *input* into the ocean between the subpolar band of 40 and 60 S. The general lack of land mass at these latitudes means that the air has largely equilibrated with the ocean in terms of both temperature and humidity and hence the sensible and latent heat fluxes are negligible. Note the same thing happens in the North Pacific but is masked in the net by the north Atlantic and the fluxes in the Kuroshio (figure 3.2e)

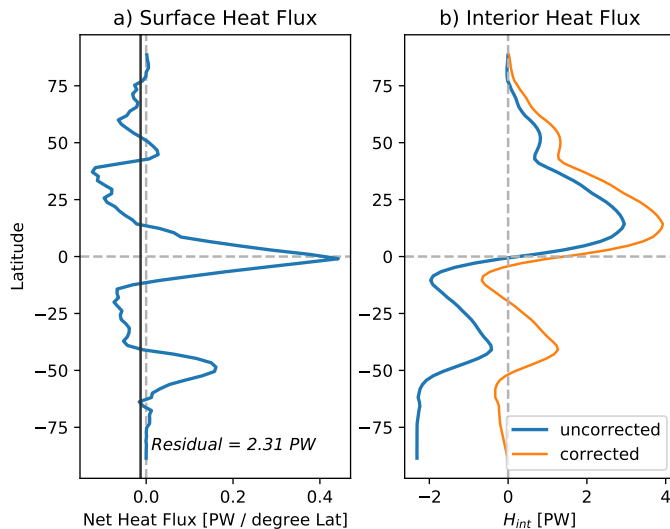


Figure 3.10: a) Globally integrated annual-mean surface heat flux per one degree of latitude. The sum of the heat flux is shown as the “residual”, and is about 4% of the net incoming radiation. b) the inferred heat transport carried by the ocean, computed by integrating the curve in (a). Negative heatflux means heat is flowing from north to south. Theoretically, this should come to zero, so this curve requires correction (orange curve is a crude correction).

As noted above, the 2.3 PW of residual heating is likely a result of error in the estimate rather than a robust idea of how fast the ocean is heating. 2.3 PW is 4% of the total incoming flux, and the estimates likely have more than 4% error in total, so the net flux is not significantly different from zero. Note that the estimate above was 1.2 PW, so the two estimates are not consistent, but they are both small compared to the total fluxes and expected errors.

On moderate time scales it is often helpful to think of the ocean as being in steady-state, so in this case it is helpful to think of the ocean as not changing temperature. On the other hand, if there is a net flux of heat into the ocean at some latitude, and a net flux out at another latitude, either the ocean where there is a net heatflux in is getting

warmer and warmer, or it is transporting heat to the region where it loses heat, as sketched in figure 3.11.

Extending this idea to many boxes is straight forward, but we have to remember that the boxes have a heat source both at the surface, and an interior flux on both sides of the box. (figure 3.12). If we assume steady state and that these heat fluxes balance, then we have $H_{in}^{(N)} = H_{in}^{(N-1)} + H_S^{(N)}$ where the bracketed superscripts refer to the cell number. Surface fluxes are defined as positive into the ocean, and interior fluxes are defined as positive if they are to the north. We can also write this as $\Delta H_{in}^{(N)} = H_S^{(N)}$ and we can see that if we have a positive heat flux into the ocean, then the northward heat flux must grow. Because this relation starts at a pole, where the interior heat flux is zero, the interior heatflux at any given latitude is the sum of the surface fluxes to the south of that latitude:

$$H_{in}^{(N)} = \sum_{i=0}^N H_S^{(i)} \quad (3.1)$$

This gives us the curve in figure 3.10b, which shows northward heat-flux in the northern hemisphere and southwards in the southern hemisphere. Again, there is a residual heat flux, so this procedure does not yield a completely sensible curve with large southwards heatflux at the southernmost ocean cell, so a correction is applied that removes the mean flux.

Direct estimates of the interior fluxes have also been made from ocean-based measurements, and a number of these are compared in figure 3.13. The level of variability in these estimates indicates the difficulties in making these types of estimates, even if the patterns make the sense.

If we look at these fluxes by ocean basin, then there are substantial differences between the Atlantic ocean and the Pacific/Indian Oceans in terms of the structure of the heat flux. In the Atlantic ocean the heat flux is everywhere north. This is because the heat flux at the equator is relatively weak, and there is substantial heat flux in the south Atlantic.

We have said nothing about what drives these heat fluxes, however, the basic idea is that warm water is less dense, and hence more buoyant, whereas cold water is less buoyant. Just like in an estuary, this creates north-south pressure gradients that drive the warm water northwards.

3.2 Surface freshwater fluxes

The ocean has a large reservoir of salinity compared to the rate of inputs (from rivers) and outputs (from forming **evaporites**), to the

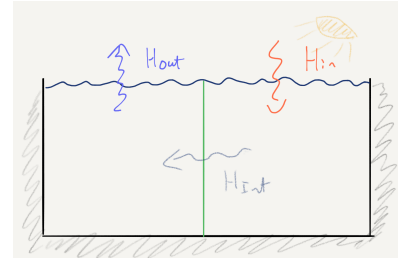


Figure 3.11: Schematic two-box representation where the right-hand box is being heated, the left-hand box cooled, and a heat flux in the interior of the box is assumed to be from warm to cold.

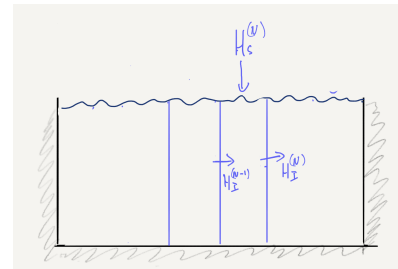


Figure 3.12: Schematic of the multi-box extension. Each box receives a surface heat flux $H_S^{(N)}$, and passes an interior flux $H_I^{(N)}$ to its northward neighbor; the interior flux entering box N from the south is $H_I^{(N-1)}$.

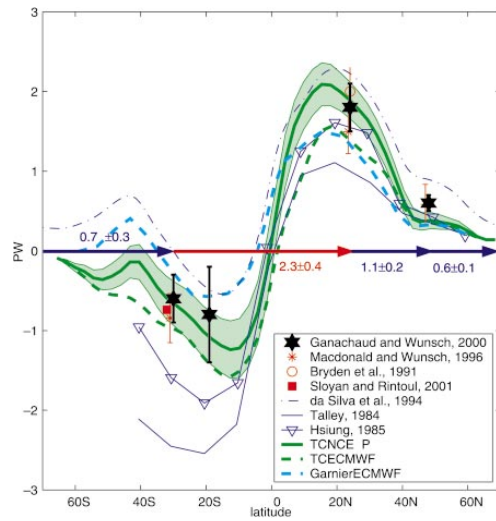


Figure 3.13: Estimates of heat transport by the world's oceans [?].

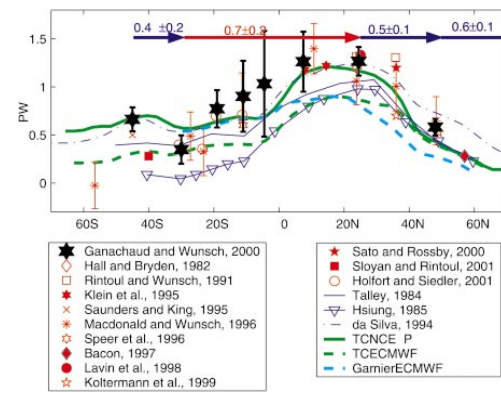
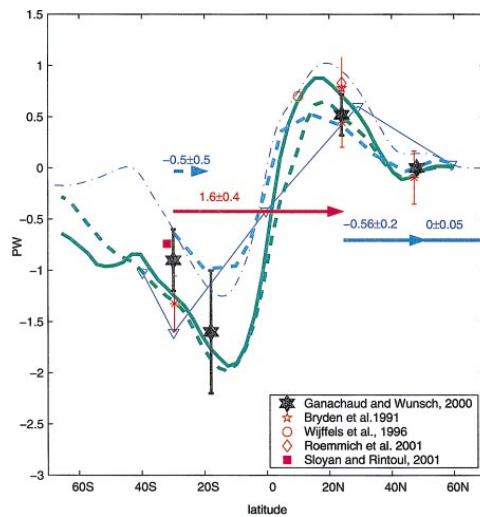


Figure 3.14: Estimates of heat transport in the Atlantic (upper panel) and Pacific and Indian oceans (lower panel)[?].



point that the mass of salt in the ocean can be regarded as a constant on the timescale of decades to centuries. Given the volume of the ocean is approximately $1.335 \times 10^{18} \text{ m}^3$, and a typical density is 1030 kg m^{-3} , and salinity is $S \approx 35 \text{ ppt}$, the mass of salt in the oceans is approximately $4.5 \times 10^{19} \text{ kg}$. Conversely, rivers deliver on the order of $3 \times 10^{12} \text{ kg}$ of salt per year, so, in order to increase ocean salinity by one part per thousand, it would take more than 450,000 years of salt input. Given that much of this input is balanced by output to forming evaporites, we tend to think of the salt balance as not changing over sub-geological timescales.

Of course this doesn't mean that salinity is constant in the ocean! Salinity is almost zero in rivers, and is zero in rainfall and at locations where these meet the ocean the salinity drops. Conversely if there is evaporation and ice formation, then salinity is left behind and the ocean is saltier in those locations. When we think about the large-scale budgets then, we often think of them as freshwater budgets, with the understanding that these are also salinity budgets.

3.2.1 *Evaporation and precipitation patterns*

As noted above, evaporation is complicated to estimate because it depends on the temperature difference of the ocean and atmosphere, the humidity of the atmosphere, and all of those parameters depend on the turbulence near the boundary between the two fluids. Many of the spatial patterns of evaporation are the same as those for latent heat loss (figure 3.15a), with evaporation largest where cold dry air passes over warm ocean currents.

Freshwater re-enters the ocean via precipitation and rivers. The precipitation patterns are largely driven by the **Hadley cell** circulation of the atmosphere, with large upward motion at the equator driving strong rainfall there (figure 3.15). The subpolar latitudes see more rainfall as well because there is also rising air in the midlatitude cell, centred at about 60 degrees. Dry air predominates at the subtropical latitudes where this air tends to be subsiding.

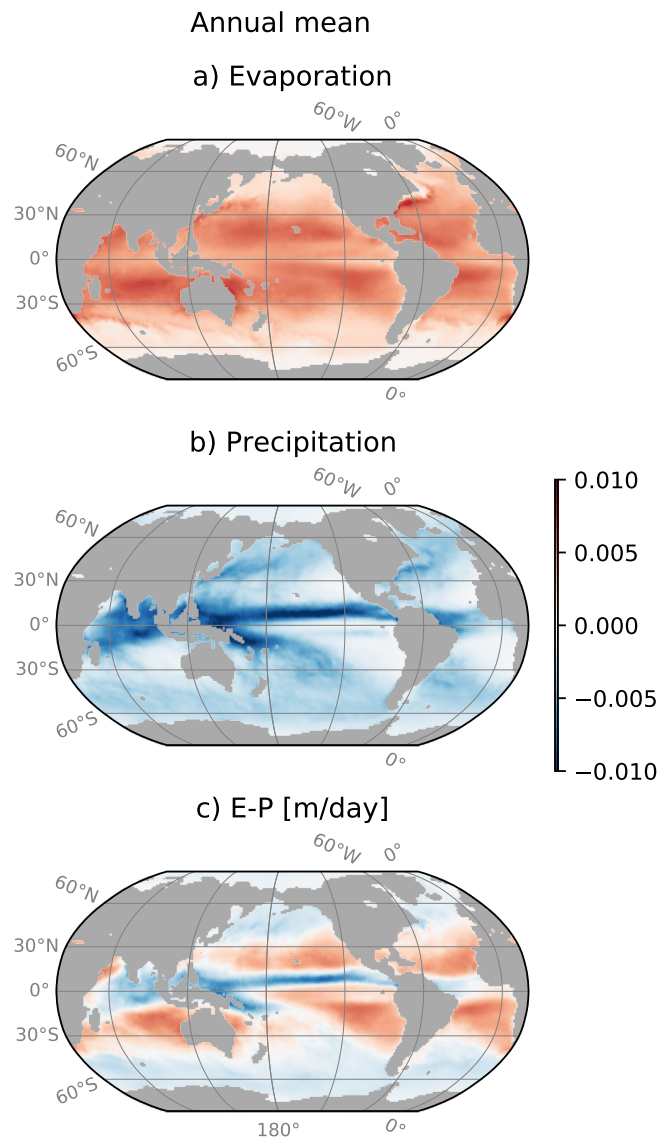


Figure 3.15: Annual average evaporation and precipitation (from ECMWF-CERA-20c re-analysis).

The net pattern of evaporation minus precipitation generally says that moisture is removed from the oceans in a band from approximately 5 degrees to 35 degrees, and in the net, precipitates near the equator and north of 40 degrees. As with heat, this necessitates a transport of freshwater from places of excess precipitation to places of excess evaporation if the ocean is to stay in approximate steady state (figure 3.16).

3.3 Conservation of mass, contaminants (and heat) in the ocean

Having described these surface fluxes qualitatively, we now turn to quantifying how heat and freshwater are transported once they are inside the ocean. We saw in the previous section that heat and freshwater can enter and leave the ocean at the surface. We also *inferred* that it must be transported from sources to sinks at the surface if the ocean is believed to be in a quasi-steady state. We call this an *inverse estimate* because we infer the inner workings of the ocean from its external forcing. In this chapter we start grappling with *how* that transport takes place, and how we might quantify and measure it. There are actually *many* processes involved in the transport of heat from the equator poleward, and trying to understand them all is one of the main challenges of oceanography. Given that, this chapter only gives some tools to start understanding fluxes and transports, and as such uses some simple examples.

Note that temperature, (or heat), for instance, can change at a location away from the surface in two possible ways. Currents can bring water with a different temperature from somewhere else; we call this an *advective flux*. Conversely the water can exchange heat via mixing, both at the large scale (stirring) and ultimately at the small scale (molecular diffusion). We call this a *diffusive flux*.

3.3.1 Conservation of water mass and volume

Water mass is conserved in a body of water, and because water is almost incompressible, we can also say that volume is conserved. To see this, consider a lake with two rivers flowing into it and one river flowing out (figure 3.17). The mass of water in the lake is $M = \bar{\rho}V$ where $\bar{\rho}$ is the average density of water and V is the volume of water in the lake. The rate of change of mass in the lake is then given by the difference between the inflowing and outflowing mass transports:

$$\frac{dM}{dt} = -T_M(1) - T_M(2) \quad (3.2)$$

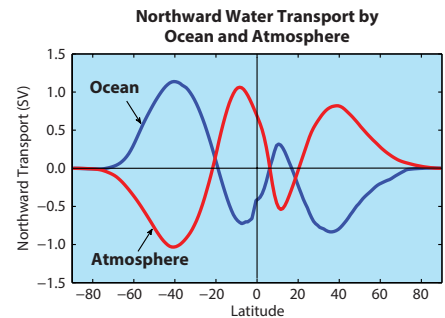


Figure 3.16: Net annual fresh-water transport [from ?, Fig. 5]. The ocean curve is estimates by integrating the net surface flux curve integrated from figure 3.15, as described above for heat. The atmospheric curve is estimated from the ERA moisture transport simulations.

where $T_M(i)$ is positive if it is flowing out of the lake, and negative if it is flowing into the lake. All terms here are in units of kg s^{-1} . We can estimate the mass transport in terms of the volumetric flow rate Q and the density ρ of the water, so that $T_M = Q\rho$:

$$\frac{dM}{dt} = -Q_1\rho_1 - Q_2\rho_2 \quad (3.3)$$

where again Q_i is positive if flowing *out* of the lake.

The conservation of mass is exact, but the density of water does not vary much in the ocean (less than 4%), so to very good approximation we can rewrite this equation in terms of volume transports:

$$\frac{dV}{dt} = -Q_1 - Q_2 \quad (3.4)$$

This is called the **Incompressible flow** approximation, and it is very useful in oceanography because it allows us to ignore the small variations in density and just think about volume transports. The only time we *cannot* assume density variations don't matter is when we are considering pressure gradients as discussed in *Chapter 2*. A little more justification is given in Box 3.3.1, but assuming incompressibility of the fluid makes the mathematics significantly easier. Of course one can always be more precise if needed. Overall, if we have many inlets and outlets, we can sum them up to get the general volume conservation equation:

$$\frac{dV}{dt} = -\sum_i Q_i \quad (3.5)$$

We usually estimate Q_i transports using a measured velocity u , and multiplying by a cross-sectional area dA . Here dA is perpendicular to the surface of the volume and again points outwards from the volume. The velocity, u is the component of velocity perpendicular to the surface, and is either positive or negative as makes sense for the flow direction.

$$\frac{dV}{dt} = -\sum_i u_i dA_i \quad (3.6)$$

Note that for a flow sketched as in figure 3.18 both u_1 and u_2 are positive, but the flow is a gain of volume through the river labeled dA_1 , and a loss from the river labeled dA_2 . Hence we have to be careful to define the direction of positive flow, and then use that definition consistently.

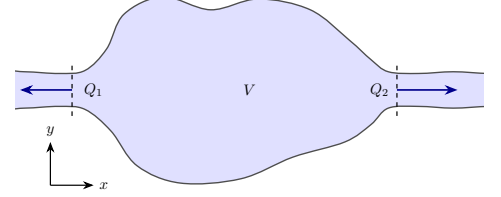


Figure 3.17: Schematic of a lake with two rivers, and the volume transport labeled with the direction that is positive.

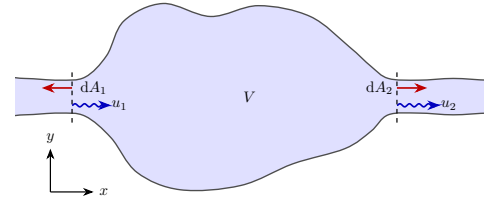


Figure 3.18: Schematic of a lake with two rivers. The cross sections of the rivers are labeled dA_1 and dA_2 , and point out of the lake, since the lake is the volume we are considering. The velocities u_1 and u_2 are the components of velocity perpendicular to the cross sections, and are positive in the direction of the arrows. Note that u_1 is a gain of volume, and u_2 is a loss of volume.

Box 3.3.1: More justification for dropping ρ

To see why its a good approximation to drop the density from the mass equation and just conserve volume, lets say that $\rho_{in} = \bar{\rho} + \rho'_1$ and $\rho_{out} = \bar{\rho} + \rho'_2$. More formally, we can expand equation (3.3) using the chain rule:

$$V \frac{d}{dt} (\bar{\rho}) + \bar{\rho} \frac{d}{dt} (V) = \bar{\rho} Q_{in} - \bar{\rho} Q_{out} + \rho'_1 Q_{in} - \rho'_2 Q_{out} \quad (3.7)$$

or, dividing by $\bar{\rho}$:

$$V \frac{1}{\bar{\rho}} \frac{d}{dt} (\bar{\rho}) + \frac{d}{dt} (V) = Q_{in} - Q_{out} + \frac{\rho'_1}{\bar{\rho}} Q_{in} - \frac{\rho'_2}{\bar{\rho}} Q_{out} \quad (3.8)$$

A very large density variation in the ocean would be 1%, so all the terms multiplied by $\frac{1}{\bar{\rho}}$ are on order of 1% of the terms that have not been multiplied by this factor. Given that 1% would be a very good estimate of any of the other terms, in a volume budget we usually ignore the small error and use equation (3.5).

As an example, suppose $Q_{in} = 1 \times 10^4 \text{ m}^3 \text{ s}^{-1}$ and $\rho_{in} = 1000 \text{ kg m}^{-3}$, and $Q_{out} = 1.1 \times 10^4$ and $\rho_{out} = 1005$, and $\bar{\rho} = 1002.5$, and $V = 10^{10} \text{ m}^3$. We see that the terms are

$$\frac{1}{\bar{\rho}} \frac{dM}{dt} = 10^3 + 2.5 \left[\text{m}^3 \text{ s}^{-1} \right] \quad (3.9)$$

and that the second term is far smaller than the first.

The left hand terms can also be compared, though it is a bit more inference. The term $\frac{V}{\bar{\rho}} \frac{d}{dt} (\bar{\rho})$ can be estimated from $\delta\rho = \rho_{out} - \rho_{in} = 5 \text{ kg m}^{-3}$ and $\delta t = Q/V$ the residence time of the lake, so this term is approximately $Q\delta\rho/\bar{\rho} \approx 10^4 * 5/(1002.5) = 50 \text{ m}^3 \text{ s}^{-1}$, which is still much smaller than the other terms in the equation.

3.3.2 Advective transport and flux

We have already discussed advective transports in the context of estuarine flows and salt transport, but lets review here. A simple example is a river with a contaminant in it. Often we will quantify a contaminant in terms of its concentration, C , which is the mass of contaminant per volume of water, so the units of C are g m^{-3} . Sometimes we will do a fractional concentration, like parts per thousand (eg Salt), and sometimes we want heat, which has units of J m^{-3} , but the concept is the same.

So in our lake example, imagine we have a contaminant at river 1

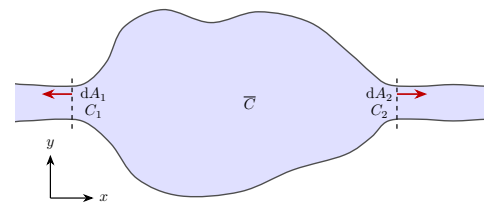


Figure 3.19: Lake with contaminant coming in from the two rivers.

with concentration C_1 , and at river 2 with concentration C_2 . The rate of change of the mass of contaminant in the lake is then given by

$$\frac{dM_C}{dt} = -Q_1C_1 - Q_2C_2 \quad (3.10)$$

where again, Q_i is positive if flowing *out* of the lake. Or if we define the mean concentration in the lake as $\bar{C} = M_C/V$, then we can write this as

$$V \frac{d\bar{C}}{dt} = -Q_1C_1 - Q_2C_2 \quad (3.11)$$

If the volume of the lake is constant, then we can write as

$$V \frac{d\bar{C}}{dt} = -Q_1C_1 - Q_2C_2 \quad (3.12)$$

and arrive at a rate of change of concentration in the lake.

The term Q_iC_i is the advective transport of the contaminant, and has units of g s^{-1} . It is the mass of contaminant per time that is being transported by the flow. As for the volume transport, we sometimes estimate this from the velocity and cross-sectional area of the flow, so that $Q_i = u_i dA_i$, and then the *advective transport* is $F_C = dA_i u_i C_i$. It is often useful to think of the transport per unit area, which we call the *advective flux*, $u_i C_i$, and has units of $\text{g s}^{-1} \text{m}^{-2}$. If the advective flux is specified instead of the transport, we need to multiply by the cross-sectional area to get the transport, so that $F_C = dA_i u_i C_i$. More generally, if we have many inlets and outlets, we would say

$$\frac{dV\bar{C}}{dt} = -\sum_i Q_i C_i = -\sum_i u_i dA_i C_i \quad (3.13)$$

Box 3.3.2: Flux gradients

The integrated version of mass conservation is often the most practically useful. However, it is often useful to think about the local rate of change of concentration at a point in space. In this case we can think about the fluxes in and out of a small box, and the rate of change of concentration in that box. If we have a box with volume $dV = dx dy dz$, then the rate of change of mass in that box is given by

$$\frac{dC}{dt} = -\frac{1}{dV} \sum_i u_i dA_i C_i \quad (3.14)$$

If we take the limit as $dV \rightarrow 0$, then we can write this as a divergence of the fluxes in and out of the box:

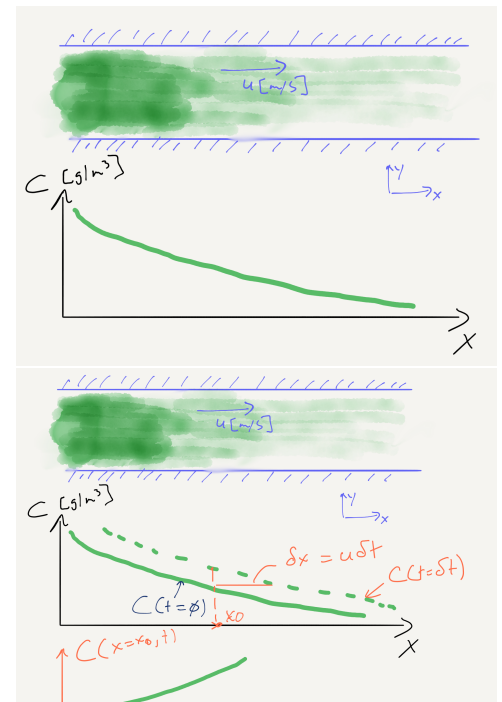
$$\frac{dC}{dt} = -\frac{\partial}{\partial x}(uC) - \frac{\partial}{\partial y}(vC) - \frac{\partial}{\partial z}(wC) \quad (3.15)$$

with flux components uC , vC , and wC in the x , y , and z directions respectively, and we have used *partial derivatives* to indicate that the derivatives are taken with respect to the spatial coordinates, and not time. If the flow is incompressible, $\partial_x u + \partial_y v + \partial_z w = 0$, then we can also write this as

$$\frac{\partial C}{\partial t} = -u \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial y} - w \frac{\partial C}{\partial z} \quad (3.16)$$

This is the form most often used in the literature, and tells us that the rate of change of concentration at a point is given by the flow speed in each direction multiplied by the gradient in concentration in that direction.

To see this consider a river with a contaminant C in it, measured with units of g m^{-3} where the gradient decreases downstream (figure 3.20), and therefore if $u > 0$, $dC/dx < 0$. If we stand at some point on the river bank, say $x = x_0$, and measure the concentration over time, clearly, the concentration will be seen to be increasing. Hopefully it is also clear that it is increasing at a rate that depends on both how fast the flow is and how strong the gradient is. If there were no gradient, then the concentration at x_0 would be constant, whereas if the gradient is strong the change will be faster. Indeed $dC/dt = -u dC/dx > 0$.



3.4 Diffusive fluxes and transports

The other way that the concentration of a contaminant can change in a fluid is for the concentration to diffuse (or heat, or momentum). The diffusion of a contaminant is at the molecular level due to the random motion of the particles in solution (figure 3.21) which acts to homogenize the concentration of the contaminant. This means that the flux of contaminant ($\text{gm}^{-2}\text{s}^{-1}$) is from high concentration to low concentration. Note that diffusive fluxes have exactly the same units as advective fluxes, and have the same effect on the concentration of the contaminant.

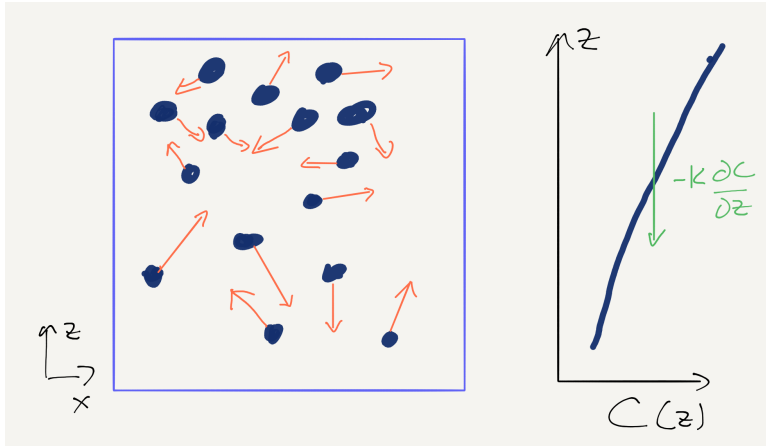


Figure 3.21: Schematic of diffusion of a contaminant. Random motions in the fluid act to homogenize the concentration, with a flux from high concentration to low concentration.

Most contaminants in fluids are assumed to obey **Fick's law of diffusion** where the diffusive flux is linearly proportional to the gradient in concentration. i.e. if there is a sharp change in the concentration, the flux is stronger. And of course the flux is always from high concentration to low concentration, so we need a minus sign:

$$F = -K_C \frac{dC}{dz} \quad (3.17)$$

where K_C is the *diffusion co-efficient*, and has units of $\text{m}^2 \text{s}^{-1}$. The sign of this flux indicates whether the flux is in the positive z -direction, or the negative. So in figure 3.21b, the $dC/dz > 0$ so the net diffusive flux of C is downwards.

The Diffusion co-efficient K varies by the contaminant, or if temperature or momentum are being considered. For salinity $K \sim 10^{-9} \text{ m}^2 \text{s}^{-1}$, where the exact co-efficient depends on the salinity and temperature of the water. For temperature, it is two orders of magnitude larger $K \sim 10^{-7} \text{ m}^2 \text{s}^{-1}$, because the transfer of heat does not require molecules to trade position, but rather for vibrational energy (heat) to be transferred, which is a more efficient process.

Of course, if we have a two- or three-dimensional distribution of the contaminant, then we need to consider the gradients in each direction. i.e. going back to the example of the river, the diffusive flux would be in the same direction as flow, since the high concentration is coming from upstream, so in this case the flux is $-KdC/dx$.

3.5 Net flux: advection plus diffusion

The total rate of change in a volume is given by the sum of the advective and diffusive transports into the volume. In the case of a contaminant, we need to multiply by cross sectional areas to get the transport of the contaminant, so that $F = -KdC/dx + uC$. If we have many inlets and outlets, then we can sum them up to get the general diffusive transport equation:

$$\frac{d}{dt}(CV) = -\sum_i Q_i C_i + \sum_i \left(dA_i K \frac{dC}{dn} \right) = -\sum_i dA_i \left(u_i C_i - K \frac{dC}{dn} \right) \quad (3.18)$$

where dA_i is the area element the fluxes are passing through, and dC/dn is a gradient, defined as positive *out of* the volume in the direction perpendicular to the area element dA_i (figure 3.22).

Note the sign of the diffusive transport. The trick is that both dA_i and dC/dn are measured in the same direction — the one pointing *out of* the lake. Lets look at river 1 in figure 3.22, where out of the lake is to the left. If the concentration in the river is less than that in the lake, then dC/dx is positive, but dC/dn is negative, because n points to the left. The term $dA_1 K dC/dn$ is therefore negative, i.e. a loss of contaminant from the lake. That is what we should expect: C is higher in the lake than in the river, so it diffuses out.

Now river 2, where out of the lake is to the right, so $dC/dn = dC/dx$. If the concentration in the lake is again greater than in the river, then dC/dn is negative, and $dA_2 K dC/dn$ is negative — again a loss. So the two faces point in opposite directions, but the sign comes out the same, because the gradient is always measured in the direction the area element points.

Of course, if the concentration in the lake is less than in the river, then the fluxes are reversed, and the lake gains contaminant from the river with the greater flux (try this to make sure it makes sense).

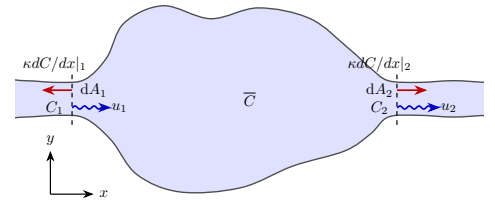


Figure 3.22: Information needed for mass budget across a surface leading out of a lake.

Box 3.5.1: Vector form of equation (3.18)

Again, if we know a little vector calculus, we can rewrite equation (3.18) as

$$\frac{d}{dt} \int_V C dV = - \oint_A (C\mathbf{u} - K\nabla C) \cdot d\mathbf{A}, \quad (3.19)$$

where $C\mathbf{u}$ is the advective flux, and $-K\nabla C$ is the diffusive flux, both of which are vector quantities.

Worked example: contaminant budget for a lake Consider the lake in figure 3.22. It has a surface area of 10 km^2 and a mean depth of 10 m , so its volume is $V = 1 \times 10^8 \text{ m}^3$. River 1 enters from the west and river 2 drains it to the east, both carrying $Q = 10 \text{ m}^3 \text{ s}^{-1}$, so the lake level is steady. The lake is well mixed at $\bar{C} = 5 \text{ g m}^{-3}$, and river 1 is cleaner than the lake, at $C_1 = 2 \text{ g m}^{-3}$. The mouth of river 1 has a cross section of $dA_1 = 20 \text{ m}^2$, and the concentration changes from the river value to the lake value over about 30 m . Take $K = 1 \text{ m}^2 \text{ s}^{-1}$ at the mouth. How fast is the lake initially cleaning up the contaminant?

Advection at river 1. Water comes in at $10 \text{ m}^3 \text{ s}^{-1}$ carrying 2 g of contaminant in every cubic metre, so the lake gains

$$Q_1 C_1 = 10 \times 2 = 20 \text{ g s}^{-1}. \quad (3.20)$$

Diffusion at river 1. Here n points west, out of the lake. Walking that way, C falls from 5 in the lake to 2 in the river over 30 m , so

$$\frac{dC}{dn} \approx \frac{2 - 5}{30} = -0.1 \text{ g m}^{-3} \text{ m}^{-1}, \quad (3.21)$$

and the diffusive transport is

$$dA_1 K \frac{dC}{dn} = 20 \times 1 \times (-0.1) = -2 \text{ g s}^{-1}. \quad (3.22)$$

The minus sign says this is a loss. That is what we should expect: the lake is dirtier than the river, so contaminant diffuses *up* the river, against the flow. Note that at this one face the advective transport is a gain and the diffusive transport is a loss. They do not have to point the same way.

River 2. The water leaving is lake water, so it carries the lake concentration and the advective transport is

$$-Q_2 C_2 = -10 \times 5 = -50 \text{ g s}^{-1}, \quad (3.23)$$

a loss. There is no diffusive transport here, because the water just downstream of the outlet is the same water that was in the lake — there is no gradient for diffusion to work on.

Adding up. Using equation (3.18),

$$\frac{d}{dt}(\bar{C}V) = 20 - 2 - 50 = -32 \text{ g s}^{-1}, \quad (3.24)$$

and since V is constant,

$$\frac{d\bar{C}}{dt} = \frac{-32 \text{ g s}^{-1}}{1 \times 10^8 \text{ m}^3} = -3.2 \times 10^{-7} \text{ g m}^{-3} \text{ s}^{-1}, \quad (3.25)$$

or about -0.03 g m^{-3} per day. The lake is slowly getting cleaner, which makes sense — we are flushing it with cleaner water.

Two things are worth noticing. First, the diffusive transport is only 2 g s^{-1} against 50 g s^{-1} of advection, so it is a 4% correction here. At a river mouth, advection usually wins. Second, the answer has to stop being right eventually: as $\bar{C}$ falls towards 2 g m^{-3} , the outflow carries away less and the gradient at river 1 weakens, until at $\bar{C} = C_1$ everything balances and $d\bar{C}/dt = 0$. At the rate above it would take roughly 100 days to close most of that gap, which is about the flushing/residence time of the lake, $V/Q = 10^7 \text{ s}$. That the two agree is a good sign we have not made an arithmetic error.

3.5.1 Turbulent diffusivities

Molecular diffusion can be quite slow. The diffusion equation ($\frac{dC}{dt} = K \frac{d^2C}{dx^2}$) can be solved with exponential functions (see Box 3.5.2) that represent how quickly the solution relaxes to its steady state. E.g. in figure 3.23, the temperature is fixed at two ends of an iron bar, and the initial temperature is cold, then heat will diffuse from the warm side to the cold side until a steady state is reached. In this case, the steady state is a linear temperature gradient so that $d^2T/dx^2 = 0$. How quickly this will happen is given by the exponential approach to the steady state.

We can estimate this exponential timescale, quite simply as $\mathcal{T} = L^2/K$, where L is the spatial scale. If we assume a 100-m lateral scale, and a molecular thermal diffusivity of $K = 10^{-7} \text{ m}^2 \text{ s}^{-1}$, we would get a time scale of 10^{11} seconds, or more than 3000 years. If we went all the way to the bottom of the 4000-m deep ocean, the time scale would be greater than 5 million years. All these numbers are 100 times larger for salt.

TIME SCALE OF DIFFUSION

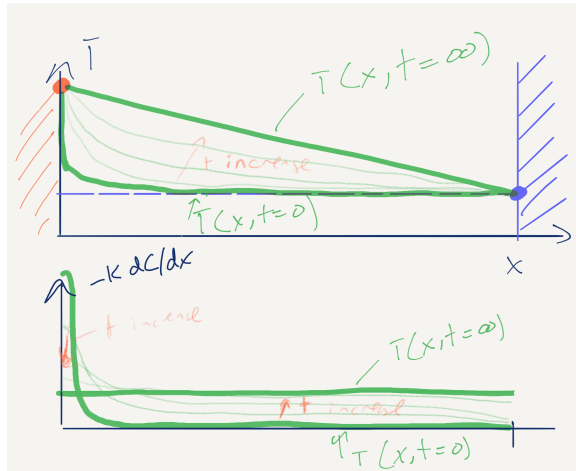


Figure 3.23: Example diffusion where temperature is fixed on the left and right side of the domain and the temperature all starts out cold. Imagine an iron bar, with one end in a fire and the other end in a pool of water. Heat diffuses in the bar from hot to cold. The steady state is a straight line between the two temperatures.

Box 3.5.2: Solutions to diffusion equation

A rigorous solution of the diffusion equation $\frac{dC}{dt} = K \frac{d^2C}{dx^2}$ needs more mathematics than we assume here, but the key result is simple: any deviation from a smooth, steady-state profile decays away on a time scale $\mathcal{T} = L^2/K$, where L is the length scale of the deviation. The same scaling describes how a localized patch of contaminant spreads out: its width grows approximately as $\sqrt{Kt}$.

3.5.2 Turbulent diffusion

Clearly, if molecular diffusivity were the only way that diffusion happened, we would just ignore these fluxes. But we saw in chapter 1 that mixing was an essential ingredient to the circulation. What happens in the real ocean is that the flow is often turbulent, and turbulence stirs the fluid, and greatly enhances the efficiency of the molecular diffusion. This is easy to see when you add milk to coffee or tea - if you do not stir the coffee, then the milk will stay in a blob. Net diffusion only happens where there are gradients in the concentration (figure 3.24). Stirring the fluid greatly increase the length of the fluid over which there is a gradient, and often sharpens the gradients. This leads to much more mixing than if the fluid is not stirred.

Stirring in the ocean is driven by a number of instability processes. *Shear instability* is one such process (figure 3.25) where the flow goes unstable because it has different velocities between layers. In the simulation in figure 3.25, the instability grows and accomplishes sig-

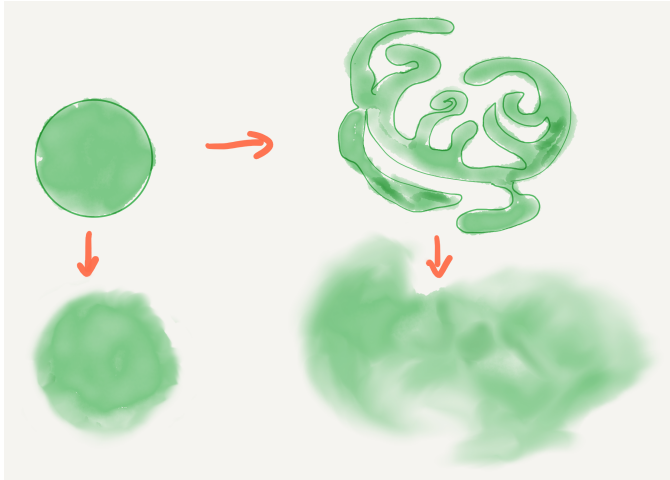


Figure 3.24: 2-D sketch of the difference between diffusion and stirring and diffusion. Of course this happens in three dimensions in the ocean.

nificant stirring before it becomes fully three-dimensional turbulence which does more stirring. The result is net mixing and thickened interface between the two layers. A natural example from Admiralty inlet shows that this type of instability is often found in nature (figure 3.26).

A second common instability is *convective instability*. This happens when dense fluid is on top of light fluid, such as happens when you boil water on the stove. A more oceanographic experiment is shown in figure 3.27 where the water is warmed from below. As it rises, there are clear instabilities, however note that these instabilities themselves are actually shear instabilities when looked at closely.

The effect of turbulence is to greatly increase the rate at which properties are fluxed across interfaces. However the flux still tends to be from regions of high concentration to regions of low concentration. Hence we will often represent these fluxes by a linear flux law. For instance, in the x -direction: $F_x = -K \frac{dC}{dx}$ where now K is a *turbulent diffusivity* rather than a molecular one. When estimated, turbulent diffusivities are *much* larger than molecular diffusivities. Estimates in the open ocean range from molecular levels to $K \sim 10^{-1} \text{ m}^2\text{s}^{-1}$ and levels can reach even higher values in coastal waters where there are strong flows. Estimating turbulence requires sophisticated instrumentation and often simplifying models, and remains a great challenge today to oceanographers. Often the approach to estimating turbulence is to relate the turbulence to external parameters that can be measured or simulated. This has lead to many schemes to estimate turbulence in models and ocean calculations.

The consequence of turbulent mixing is that it makes the importance of mixing much more apparent. If the timescale is 10^{11} s

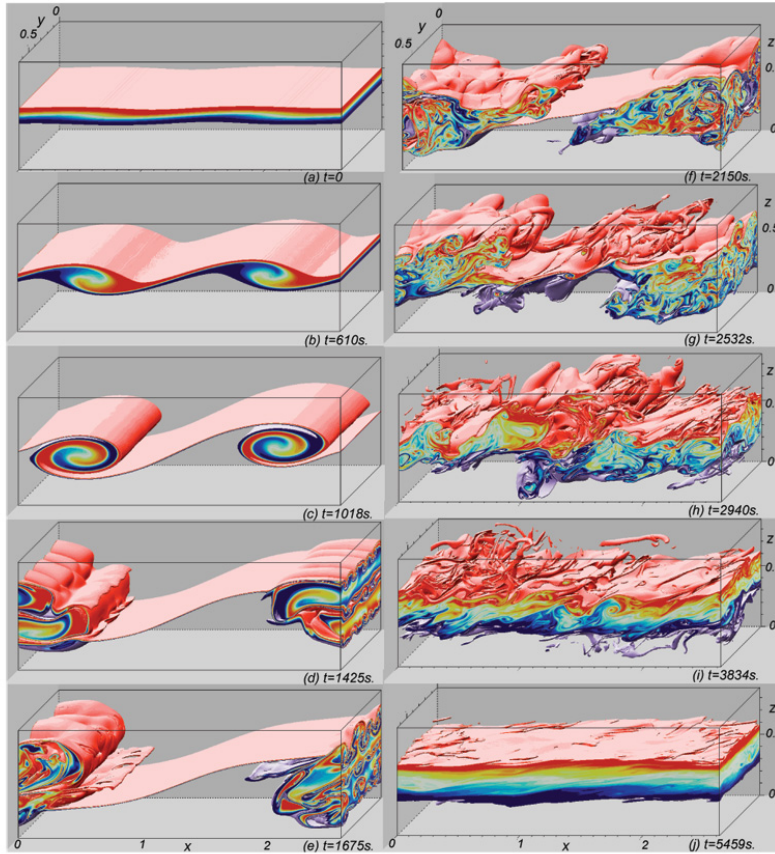


Figure 3.25: Numerical simulation of mixing in the ocean. Here a flow in the upper layer from left to right causes an instability on the temperature interface in the middle. The flow breaks down into turbulence, and the turbulence stirs the fluid driving greatly enhanced mixing. [?]

for molecular diffusion at $K = 10^{-7} \text{ m}^2 \text{ s}^{-1}$, then it is 10^7 s for $K = 10^{-3} \text{ m}^2 \text{ s}^{-1}$, or 100 days. That starts to matter for long-lasting flows.

3.6 Exercises

Diffusive fluxes The geometry and units of fluxes are a bit difficult to get clear, so let's practice them here. Consider the following table of data points:

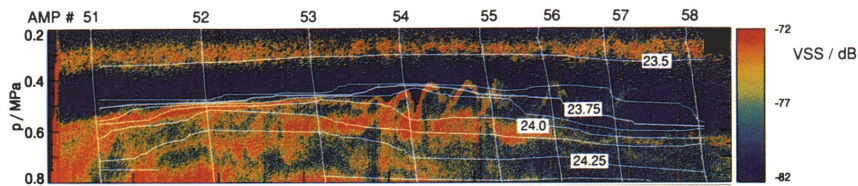


Figure 3.26: Shear instability in the ocean measured at Admiralty Inlet in Puget Sound [?].



Figure 3.27: Horizontal convection experiment where the left side of a tank is heated at the bottom, and the right hand side (not shown) is cooled. Water rises, primarily along the left hand wall.
<http://rses.anu.edu.au/gfd/AR2005/index.php?p=ar05res>

Depth [m]	Concentration Sulfur $g\ m^{-3}$
5	2
15	3
25	6
35	6
45	4
55	2

Assume these are in a lake and there is no flux of sulfur in or out of the surface or sea floor of the lake.

- Sketch the profile of Sulfur with depth.
- Doing this sort of thing abstractly becomes hard. Lets assume this profile was collected in a column of water that is 65-m deep, 1 m x 1m in the horizontal plane. Divide the column into $10 \times 1 \times 1\ m^3$ blocks centered on our measurements. Approximately, how many grams of Sulfur are there in each of the 6 blocks?
- Approximate the gradient of Sulfur concentration at each the boundaries of each of the boxes (i.e. 10 m, 20 m, etc..).
- If the turbulent diffusivity is $K = 10^{-3} m^2 s^{-1}$ what is the diffusive transport across each of the block faces? Make sure you get the direction correct. Report your result in grams/second.

- Consider any one block. You now should know the value and direction of flux (in g/s) across the upper and lower face, and you should know the total grams inside the block. If you assume the flux remains constant for 10,000 s, how many grams of Sulfur are in your cell after 10,000 s?
- Repeat for all 6 cells (hint, use a computer!) and plot the new sulfur profile after 10,000 s.
- Repeat the procedure 9 more times to get the profile after 100,000 s; (Hint: really use a computer;). Plot the sulfur profile at each timestep. Make sure to indicate an example calculation at $t > 10,000$.
- Do the changes of the profile correspond to your intuition of how diffusion works?

3.7 Example problems

The following problems are adapted from past exams, and work through the ideas above with specific numbers.

A lake's contaminant budget A lake has two rivers flowing into it and one flowing out. The lake has a surface area of 100 km^2 and a uniform depth of 10 m. At the inflow, the velocity is measured at a stream gauge to be 0.2 m s^{-1} , and the river channel there is 10 m wide and 5 m deep. At that inflow we measure a caffeine concentration of 10 g m^{-3} and a sulfur concentration of 0.9 g m^{-3} . At the outflow, the sulfur concentration is measured to be 1.0 g m^{-3} .

Assume the mass of sulfur and the mass of caffeine in the lake are both approximately in steady state, and that there is no flux of either substance through the lake's surface or bed.

- Can the volume of the lake be constant? If not, what is the rate at which the water level is rising or falling (in m/s)?
- What is the concentration of caffeine in the outflowing river?

This is really the same problem as the worked lake example above, but now the second river's transport is unknown. You need the sulfur budget to pin that transport down before the caffeine budget can be used to find the outflow concentration.

Evaporation and exchange flow at the Strait of Gibraltar The Mediterranean Sea loses freshwater at its surface at a rate of $4 \times 10^{-8} \text{ m s}^{-1}$ (a little more than 1 m/year) due to evaporation, over a surface area

of $2.5 \times 10^6 \text{ km}^2$. This evaporation is balanced by an exchange flow at the Strait of Gibraltar: relatively fresh Atlantic water flows in with $S = 36.4 \text{ psu}$, and saltier Mediterranean water flows out with $S = 38.4 \text{ psu}$.

- Using conservation of volume and conservation of salt across the strait, find the transport of Mediterranean water out through the strait, and the transport of Atlantic water in.

Diffusion into a salty vat A 10-m long pipe connects a vat of fresh water to a vat of salt water with a salinity of 25 psu. Water flows steadily through the pipe from the fresh vat toward the salty vat at 0.1 m s^{-1} , and the pipe has a cross-sectional area of 0.1 m^2 . Turbulence in the pipe mixes some of the salty water back toward the fresh end; assume the turbulent diffusivity K is the same everywhere along the pipe, and that the salinity profile in the pipe has reached a steady state.

- Sketch the set-up, including a plausible salinity profile along the length of the pipe.
- Where do you expect the salinity *gradient* to be largest – near the fresh end or the salty end?
- Justify your answer using the 1-D advective-plus-diffusive balance (equation (3.18)) applied along the pipe.

Transport across two sections of a basin Consider a 2000-m deep ocean basin, closed to the east and west by land, and two lines across it, A to the south and B to the north, that together enclose a volume of the basin. Velocity, temperature, and salinity are measured along both lines and averaged into an upper layer (0–1000 m) and a lower layer (1000–2000 m). In the upper 1000 m of line A, the transport into the enclosed volume is measured to be 2 Sv , and the average temperatures and salinities in each layer at each line are:

depth	$T_A [^\circ\text{C}]$	$S_A [\text{psu}]$	$T_B [^\circ\text{C}]$	$S_B [\text{psu}]$
0–1000 m	11	32	10	31
1000–2000 m	8	33	9	30

Assume no heat or freshwater is exchanged with the atmosphere over the enclosed volume, and that the basin is in steady state.

- What is the transport (magnitude and direction) in each of the other three layers – upper B, lower A, and lower B?
- Be explicit about your sign convention.

This extends the two-box heat-transport argument in figure 3.12 to a case with two layers, where a heat budget and a salt budget both have to be satisfied at once.

Part II

Waves and Tides

Part III

Wind-driven circulations