

JODY M. KLYMAK

INTRODUCTION TO PHYSICAL OCEANOGRAPHY

UNIVERSITY OF VICTORIA

Copyright © 2026 Jody M. Klymak

PUBLISHED BY UNIVERSITY OF VICTORIA

/

Licensed under the Apache License, Version 2.0 (the “License”); you may not use this file except in compliance with the License. You may obtain a copy of the License at <http://www.apache.org/licenses/LICENSE-2.0>. Unless required by applicable law or agreed to in writing, software distributed under the License is distributed on an “AS IS” BASIS, WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied. See the License for the specific language governing permissions and limitations under the License.

First printing, September 2026

Contents

	<i>I Overturning circulations</i>	10
1	<i>Estuaries</i>	11
1.1	<i>Pressure differences</i>	12
1.2	<i>Estuarine Flow</i>	13
1.2.1	<i>Salt-wedge estuary</i>	13
1.2.2	<i>Fjord-type estuary</i>	13
1.2.3	<i>Flood-plain estuaries</i>	14
1.2.4	<i>Time-dependence</i>	16
1.2.5	<i>Topographic blocking (mostly in fjords)</i>	19
1.3	<i>Quantifying the Circulation: Knudsen Relation</i>	19
1.3.1	<i>The Conservation of Volume</i>	20
1.3.2	<i>Conservation of salt</i>	20
1.3.3	<i>Application to Estuaries: The Knudsen Relation</i>	21
1.4	<i>Exercise</i>	23
1.5	<i>Example problems</i>	26
2	<i>Equation of State and Pressure Gradients</i>	28
2.1	<i>Temperature</i>	28
2.1.1	<i>Measurement</i>	29
2.1.2	<i>In-situ and potential temperature</i>	30
2.2	<i>Salinity</i>	31
2.2.1	<i>Measurement</i>	32

2.3	<i>Pressure</i>	32
2.3.1	<i>Measuring pressure</i>	33
2.4	<i>Calculating density</i>	34
2.4.1	<i>Density relation</i>	35
2.4.2	<i>Potential density</i>	35
2.4.3	<i>Cabbeling</i>	37
2.5	<i>Other seawater properties</i>	38
2.5.1	<i>Sound</i>	39
2.5.2	<i>Light transmission</i>	39
2.5.3	<i>Ice</i>	40
2.6	<i>Pressure Gradients</i>	40
2.6.1	<i>Hydrostatic Pressure</i>	41
2.6.2	<i>Single density fluid</i>	41
2.6.3	<i>Two-density fluid</i>	42
2.7	<i>Pressure gradients</i>	43
2.7.1	<i>Surface pressure gradients</i>	43
2.7.2	<i>Internal pressure gradients</i>	44
2.8	<i>Exercise</i>	47
2.9	<i>Example problems</i>	47
3	<i>Heat and Salt fluxes</i>	49
3.1	<i>Surface heat fluxes</i>	49
3.1.1	<i>Incoming shortwave radiation</i>	50
3.1.2	<i>Longwave radiation (back radiation)</i>	58
3.1.3	<i>Sensible heat transfer</i>	58
3.1.4	<i>Latent heat losses</i>	59
3.1.5	<i>Integrated Surface Heat Fluxes</i>	61
3.2	<i>Surface freshwater fluxes</i>	62
3.2.1	<i>Evaporation and precipitation patterns</i>	64
3.3	<i>Conservation of mass, contaminants (and heat) in the ocean</i>	66
3.3.1	<i>Conservation of water mass and volume</i>	66
3.3.2	<i>Advective transport and flux</i>	68

3.4	<i>Diffusive fluxes and transports</i>	71
3.5	<i>Net flux: advection plus diffusion</i>	72
3.5.1	<i>Turbulent diffusivities</i>	74
3.5.2	<i>Turbulent diffusion</i>	75
3.6	<i>Exercises</i>	77
3.7	<i>Example problems</i>	79
4	<i>Ocean Stratification and the Mixed Layer</i>	82
4.1	<i>Vertical structure of the ocean</i>	82
4.2	<i>Buoyancy frequency</i>	83
4.3	<i>Overturns</i>	84
4.4	<i>The surface mixed layer</i>	85
4.4.1	<i>Seasonal and diurnal cycles</i>	88
4.4.2	<i>Global mixed layer depths</i>	90
4.5	<i>Restratification of the mixed layer</i>	91
4.6	<i>What maintains the deep stratification?</i>	91
4.7	<i>Exercises</i>	91
5	<i>Ocean water masses</i>	94
5.1	<i>Surface distributions of temperature and salinity</i>	95
5.2	<i>Meridional sections</i>	96
5.3	<i>Vertical structure and θ-S diagrams</i>	99
5.4	<i>Summary</i>	105
5.5	<i>Exercises</i>	107
5.6	<i>Example problems</i>	108
6	<i>Thermohaline overturning circulation</i>	110
6.1	<i>Inferring the overturning circulation from other tracers</i>	111
6.1.1	<i>Bio-active tracers</i>	111
6.1.2	<i>Radioactive and tracers from the crust</i>	114
6.1.3	<i>Man-made tracers</i>	117

6.2	<i>Deep water formation processes</i>	121
6.2.1	<i>Antarctic Bottom Water</i>	121
6.2.2	<i>North Atlantic Deep Water</i>	122
6.3	<i>Inferring the strength of the overturning circulation from data</i>	123
6.3.1	<i>How to make an inverse calculation</i>	127
6.3.2	<i>Inverse results</i>	130
6.4	<i>Theory of overturning circulation and Sandstrom's theorem</i>	133
6.5	<i>Exercises</i>	136
6.6	<i>Example problems</i>	137
7	<i>Climate Change; The Effect on the Overturning Circulation and Sea Level</i>	140
7.1	<i>Effect of melting on the overturning circulation</i>	140
7.2	<i>Sea level change</i>	144
7.2.1	<i>Land-fast ice</i>	145
7.2.2	<i>Thermal expansion (steric change)</i>	146
7.3	<i>Future sea level rise</i>	146
7.4	<i>Other reading</i>	147
II	<i>Waves and Tides</i>	150
8	<i>Surface gravity waves</i>	151
8.1	<i>Characterizing waves</i>	152
8.2	<i>Causes</i>	153
8.3	<i>Dispersion relation, phase speed, and particle motion</i>	155
8.4	<i>Orbital motions: velocity of water parcels</i>	156
8.5	<i>Group speed</i>	158
8.6	<i>Energy conservation</i>	163
8.7	<i>Refraction and diffraction</i>	164
8.8	<i>Non-linearity</i>	166
8.9	<i>Exercises</i>	167

9	<i>Tides</i>	170
9.1	<i>Tidal example and describing tides</i>	170
9.2	<i>Tidal forces: Gravity and centrifugal forces on co-orbiting bodies</i>	171
9.3	<i>The reason for various tidal frequencies</i>	175
9.3.1	<i>Twice a day tides</i>	176
9.3.2	<i>Daily tides (diurnal): Tides due to non-equatorial orbits</i>	178
9.3.3	<i>Longer period tides</i>	180
9.4	<i>Dynamic theory of the tides</i>	181
9.4.1	<i>Tidal Resonance</i>	181
9.4.2	<i>Frictional effects</i>	183
9.4.3	<i>Global response</i>	183
9.5	<i>Exercises</i>	185
9.6	<i>Example problems</i>	186
	 <i>III Wind-driven circulations</i>	 187
10	<i>Wind-driven circulation overview</i>	188
10.1	<i>Coastal upwelling</i>	188
10.2	<i>North Pacific observations</i>	193
11	<i>Coriolis force and Ekman Balance</i>	206
11.1	<i>Quick review of forces and apparent forces</i>	206
11.2	<i>Centrifugal and Coriolis forces</i>	208
11.2.1	<i>Centrifugal force</i>	208
11.2.2	<i>Coriolis force</i>	210
11.3	<i>Friction force</i>	214
11.3.1	<i>External stresses</i>	215
11.4	<i>Ekman balance</i>	216
11.4.1	<i>The Ekman spiral: fact and fiction</i>	218
11.5	<i>Ekman convergence and divergence</i>	220
11.6	<i>Exercises</i>	223
11.7	<i>Example problems</i>	224

12	<i>Geostrophic balance and coastal flow</i>	225
12.1	<i>Geostrophic balance</i>	225
12.1.1	<i>Thermal wind</i>	227
12.1.2	<i>Level of no motion and “dynamic height”</i>	228
12.2	<i>Coastal upwelling, complete picture</i>	229
12.2.1	<i>Internal dynamics on shelf</i>	230
12.3	<i>Exercises</i>	234
13	<i>Sverdrup balance and basin-scale circulation</i>	236
13.1	<i>Circulation</i>	237
13.2	<i>Conservation of potential vorticity</i>	238
13.3	<i>Sverdrup transport</i>	240
13.3.1	<i>Alternative derivation: Ekman pumping</i>	242
13.4	<i>Return flow</i>	244
13.5	<i>Example problems</i>	245

Foreword

These are the course notes for Descriptive Physical Oceanography, taught at the University of Victoria, School of Earth and Ocean Sciences, a 13-week undergraduate course.

Unlike many physical oceanography courses, students taking this course are not expected to have studied vector calculus or differential equations, nor are they expected to have learned classical mechanics beyond what is typically offered in a first-year physics course. Many of the students have a deep interest in understanding the ocean and often approach the course with considerable intuition. The challenge of a course like this is to offer more than just a list of facts, but rather to try to give a feeling for the forces that drive the ocean, without becoming too bogged down in mathematical formalism. As such, this course tries to strike a balance between knowing about the various physical phenomena in the ocean and understanding the dynamics that give rise to them, while still giving students some practice in practical calculations.

The course is divided into three unequal parts: overturning circulations driven by lateral buoyancy differences and mixing; waves and tides; and wind-driven circulations. We start with estuaries as a nice introduction to overturning circulations, but also because the students do work in Saanich Inlet, and understanding estuaries early is very helpful.

Part I

Overturning circulations

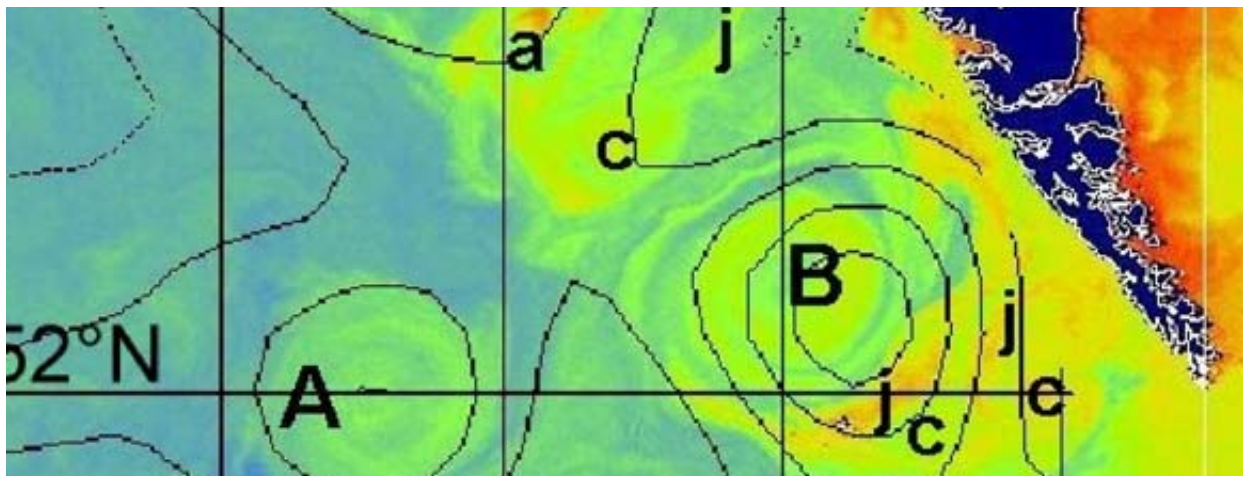
Part II

Waves and Tides

Part III

Wind-driven circulations

Geostrophic balance and coastal flow



12.1 Geostrophic balance

The second balance that involves the Coriolis force is the **Geostrophic balance**, and is when the Coriolis force is in “balance” with the pressure gradient:

$$\frac{du}{dt} = -\frac{1}{\rho_0} \frac{dP}{dx} + \underbrace{fv}_{\text{Ekman}} + \underbrace{\frac{1}{\rho} \frac{d\tau_{yz}}{dz}}_{\text{Ekman}} \quad (12.1)$$

$$\frac{dv}{dt} = -\frac{1}{\rho_0} \frac{dP}{dy} - \underbrace{fu}_{\text{Ekman}} + \underbrace{\frac{1}{\rho} \frac{d\tau_{xz}}{dz}}_{\text{Ekman}} \quad (12.2)$$

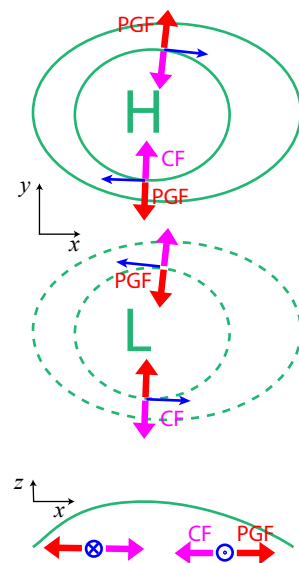


Figure 12.1: Sketch of geostrophic force-balance for a sea-level high and a sea-level low in the northern hemisphere, looking from above, and from the south for the high. The flow will be clockwise around a high and counter-clockwise.

in which case the equations of motion simplify to:

$$\begin{aligned} -\frac{1}{\rho_0} \frac{dP}{dx} &\approx -fv \\ -\frac{1}{\rho_0} \frac{dP}{dy} &\approx +fu \end{aligned}$$

In the northern hemisphere ($f > 0$) this says that water turns to the right as it moves from high pressure to low. This gives a clockwise circulation around an isolated high, or a counter-clockwise around an isolated low (figure 12.1)

Crucially, if we know the pressure gradient, we can directly calculate the geostrophic flow perpendicular to that pressure gradient from the equations above. So, suppose the high in the cover figure is 16 cm, over 100 km. The pressure difference due to this flow gives us an estimate of the velocity (assuming $f = 10^{-4} \text{ rad s}^{-1}$)

$$|u| = \left| \frac{1}{f\rho} \frac{dP}{dx} \right| = \frac{1}{f\rho} \frac{\Delta P}{\Delta x} = \frac{g}{f} \frac{\eta_C - \eta_E}{\Delta x} \approx 0.16 \text{ m s}^{-1} \quad (12.3)$$

The equation applies everywhere in the fluid, and is only made complex by the presence of water with different densities making the flow have a pressure gradient that changes with depth. An observation of this can be seen in figure 12.2, where there is a sea-surface high in the middle, but the pressure gradient inside the eddy is the other direction. This causes the pressure gradient force to decrease with depth, and hence the geostrophic flow decays with depth as well.

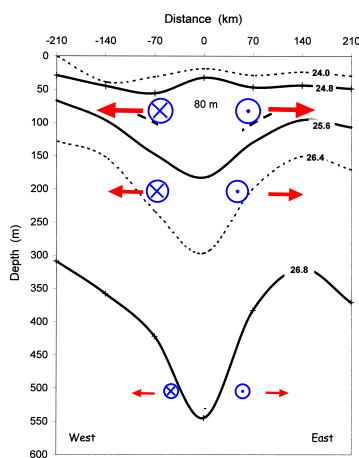


Figure 12.2: Observations of density inside an Eddy. Note how the density contours tend to counteract the high sea-surface height. Blue circles with crosses are into the page, and with dots are out of the page, and the size is sketched to be proportional to the speed of the flow.

As discussed in chapter 2, calculating the pressure gradient is simply a matter of calculating the pressure at two locations, and

differencing, where the pressure is given by:

$$P(z)/\rho_0 = \frac{1}{\rho_0} \int_z^\eta \rho g \, dz \approx g\eta + \frac{1}{\rho_0} \int_z^0 \rho g \, dz \quad (12.4)$$

So imagine one flank of the eddy, and that the eddy is made of two densities $\rho_1 < \rho_2$ (figure 12.3). In the upper layer

$$-\frac{1}{\rho_0} \frac{dP}{dx}(z = z_1) = -\frac{1}{\Delta x} g(\eta_B - \eta_A). \quad (12.5)$$

In the lower layer, things are a bit more complicated, so we just do it step by step:

$$P_A(z = z_2) = \rho_1 g(\eta_A - Z_A) + \rho_2 g(Z_A - z_2) \quad (12.6)$$

$$P_B(z = z_2) = \rho_1 g(\eta_B - Z_B) + \rho_2 g(Z_B - z_2) \quad (12.7)$$

So the total pressure gradient in the lower layer is approximated as

$$-\frac{1}{\rho_0} \frac{P_B - P_A}{\Delta x} = \overbrace{-\frac{1}{\Delta x} g(\eta_B - \eta_A)}^{\text{Surface}} - \overbrace{\frac{1}{\Delta x} g \frac{(\rho_2 - \rho_1)}{\rho_0} (Z_B - Z_A)}^{\text{Internal}} \quad (12.8)$$

According to the geostrophic balance, then, the flow in the upper layer is into the page, and the one in the bottom layer is either into the page, but more slow, or even out of the page.

12.1.1 Thermal wind

The example above makes it clear that lateral density differences can drive velocity differences in the vertical if the flow is in geostrophic balance. This can easily be extended to the continuous case with some manipulation of the momentum equations. If we take the vertical derivative of the x- and y-momentum equations we arrive at:

$$-\frac{1}{\rho_0} \frac{d^2 P}{dx dz} = -f \frac{dv}{dz} \quad (12.9)$$

$$-\frac{1}{\rho_0} \frac{d^2 P}{dy dz} = f \frac{du}{dz} \quad (12.10)$$

but $P = \int_z^\eta \rho g dz$, so we get the **thermal wind** relation.

$$-\frac{1}{\rho_0} \frac{d\rho}{dx} = -f \frac{dv}{dz} \quad (12.11)$$

$$-\frac{1}{\rho_0} \frac{d\rho}{dy} = f \frac{du}{dz} \quad (12.12)$$

so, if we know the horizontal pressure gradient, we can immediately say how the velocity changes with depth (though we do not know the absolute velocity).

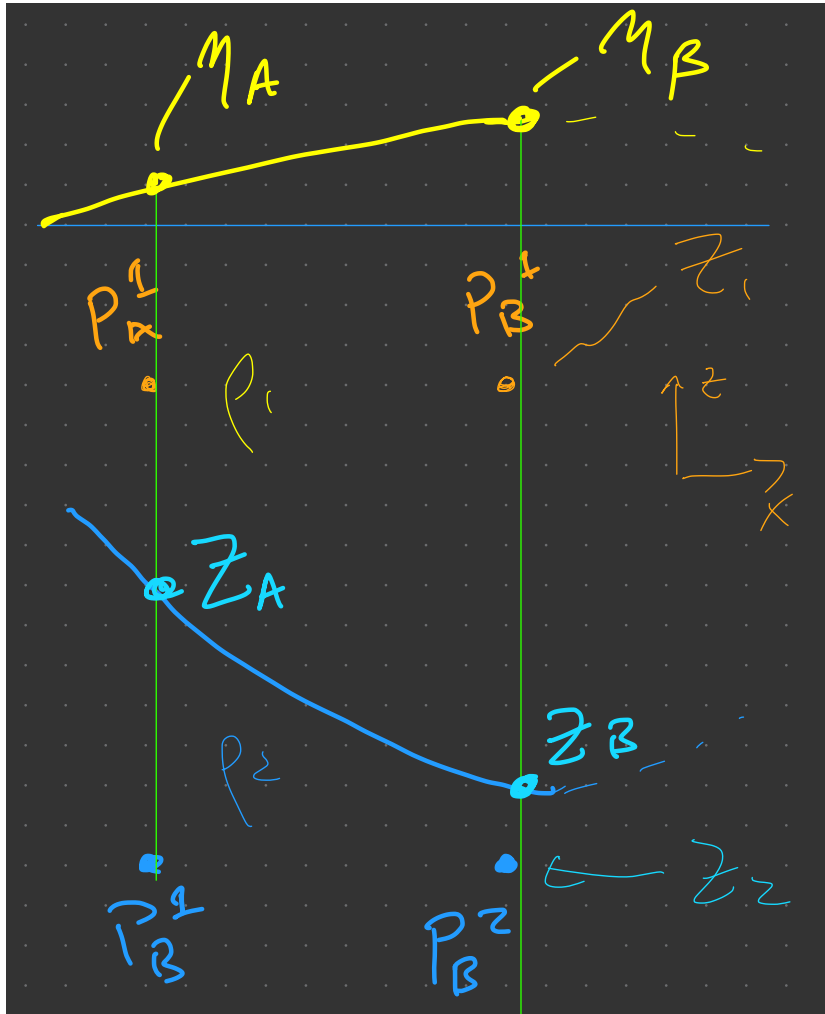


Figure 12.3: Two-layer approximation of pressure gradients in an eddy.

12.1.2 Level of no motion and “dynamic height”

Back in the pre-satellite days, all oceanographers would have had measurements of the density under the ocean, like those in figure 12.2. Estimating the sea-surface height was impossible; imagine detecting 15 cm over 100 km in an ocean that is otherwise also moving due to tides, surface waves, relative to a poorly-constrained geoid. However, oceanographers noted that most ocean currents, like the eddy, had tilted density surfaces, and that the currents at depth go to approximately zero.

Under these assumptions, oceanographers would assume that the pressure gradient at a “level of no motion” was zero, so that the geostrophic flow was zero there as well. In order to get this level of no motion the pressure gradients inferred from the interior density

measurements were assumed to be balanced by a pressure gradient from the surface at that depth. So, for instance, in the simple two-layer case the equation (12.8), the *Surface* term was equal and opposite in sign to the *Internal* term, if the deep layer was assumed to be not moving. Note that this allows an estimate of the surface pressure gradient, and using the geostrophic balance, the surface velocities. This surface pressure gradient is of course supplied by the sealevel interface being tilted, and calculating these gradients is like calculating a “dynamic height” of the ocean.

An example in figure 12.4, calculated in the 1980s looks strikingly similar to the sea surface heights in chapter 10. In particular, where there is Ekman convergence in the subtropics there is a high of up to 2 m, and a low in the sub-polar regions where there is Ekman divergence. The highs and lows are displaced to the west sides of the basins. The steepest change in dynamic height correspond to the strongest currents, again largely concentrated on the west side of the basins, and associated with the Gulf Stream, Kurishio, East Australian current etc.

Just to summarize again how this is done:

1. An interior pressure gradient is calculated at two “stations” as a function of depth using the density data from CTDs or other measurements.
2. At a “depth of no motion” the pressure gradient is assumed to be zero, so a surface pressure gradient is added to the two points to make that be the case.
3. If there are multiple stations in a line the surface pressure gradient is integrated to make a dynamic height with an arbitrary zero at some location.

We will come back to *why* the dynamic height looks the way it does in the next chapter.

12.2 Coastal upwelling, complete picture

Using the Ekman and geostrophic balances we are now in a position to explain the coastal upwelling example in detail (i.e. figure 10.3, one snapshot in detail in figure 12.5).

First, the wind starts to blow towards the south; this sets up a southwards windstress τ_y^w (figure 12.6, red circle). When this flow is in a steady-state Ekman balance there will be an equal and opposite Coriolis force to the north (figure 12.6, magenta circle). That implies

an offshore Ekman transport to the west given by

$$u_{Ek}D = \frac{\tau_y^w}{\rho_0 f} \quad (12.13)$$

The offshore Ekman transport drops the sealevel at the coast (figure 12.7, surface interface). This creates a surface pressure gradient force that acts to push water back towards the coast (figure 12.7, red arrow). Again, in *steady state* geostrophic balance, the pressure gradient force is balanced by a Coriolis force offshore, which must be generated by a geostrophic velocity to the south. Note that this evolves in time with the deep water *initially* moving onshore, but then being turned south (to the right) by the Coriolis force.

At this point the flow cannot be in steady state because we are transporting water offshore in the surface Ekman layer, but not having any onshore transport to replace it. Without this return flow, the sea surface will continue to increase its tilt. The return flow happens in the bottom Ekman layer. The interior geostrophic flow moves south, so there is a corresponding bottom stress to the north τ_y^B (figure 12.2, red circle at bottom). In a steady Ekman balance, this implies a Coriolis force to the south, which implies a net Ekman transport onshore (figure 12.2, blue arrows, bottom), given by

$$u_{Ek}D = \frac{\tau_y^B}{\rho_0 f} \quad (12.14)$$

The offshore Ekman transport drops the sealevel at the coast (figure 12.2)

When is the whole flow balanced? When $\tau^B = \tau^W$ so that the transport in the bottom Ekman layer is the same as the transport in the surface Ekman layer. So, if we know the wind stress, and have a good estimate of the bottom **drag co-efficient**, $C_D \approx 10^{-3}$, then we can guess what the mean geostrophic velocity is, and hence the size of the sea surface tilt. For the Oregon shelf, $\tau_y^W \approx -0.15 \text{ N m}^{-2}$ (minus means to the south). Then means that the geostrophic velocity $v_g \approx 0.35 \text{ m s}^{-1}$ in order for $\rho_0 C_D |v_g|^2$ to be large enough to provide the bottom stress to match. This is approximately the same as the observations near the bottom.

We can further calculate the sea-surface height drop across the 20-km width of the shelf from $g \frac{d\eta}{dx} = f v_g$ or $\Delta\eta = 0.08 \text{ m}$ across the shelf.

12.2.1 Internal dynamics on shelf

The final thing we notice about the shelf observations is that the flow is slower towards the sea-floor than at the surface. This is also largely

explained by the geostrophic balance. Note that the isopycnals tilt up towards the coast, counteracting the surface pressure gradient (also sketched in figure 12.8). This means that the surface tilt is actually larger than we calculated above if the bottom velocity is to remain 0.38 m s^{-1} , leading to faster velocities near the surface.

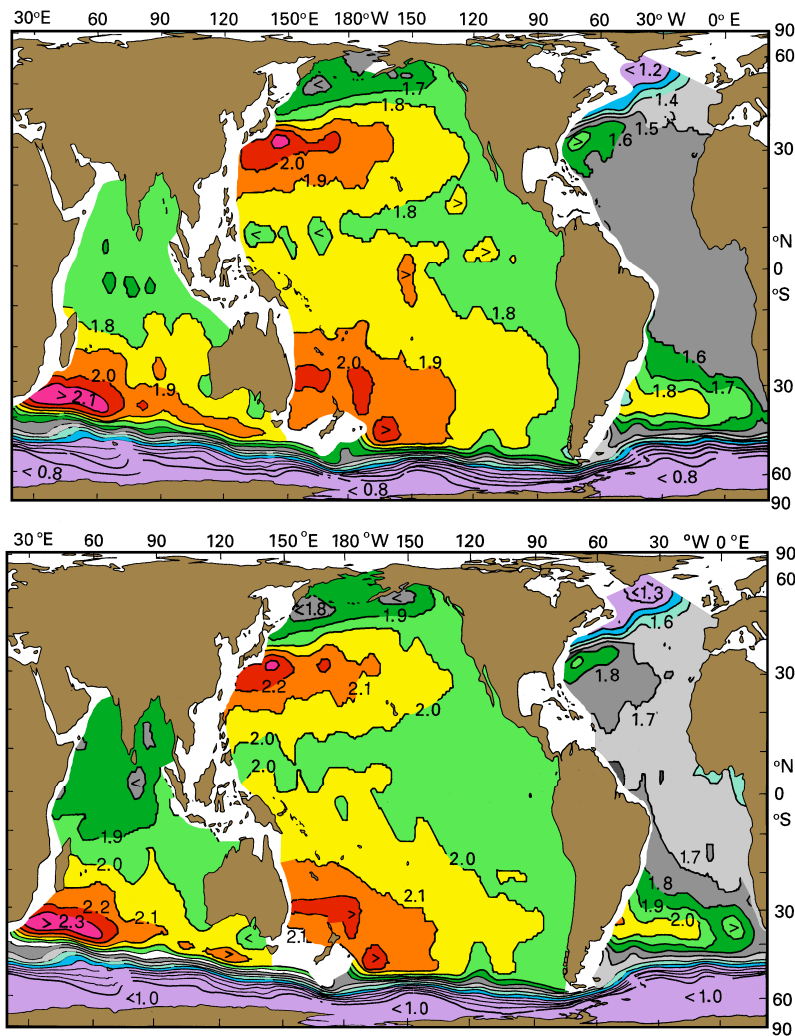


Figure 12.4: Dynamic height with a level of no motion at 1500 m (top) and 2500 m (bottom). Note these are quite similar, strongly indicative that the dynamics are largely above these depths.

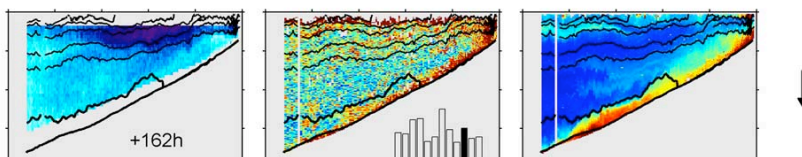


Figure 12.5: Data from figure 10.3.

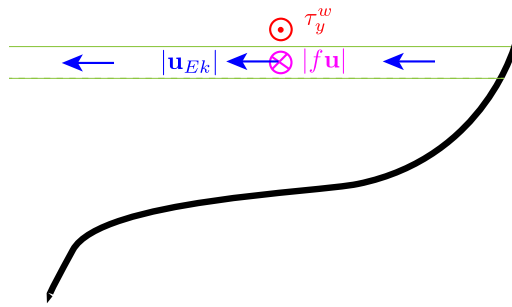


Figure 12.6: On a continental slope, initially at rest, a wind stress blows from north to south (dotted red circle)

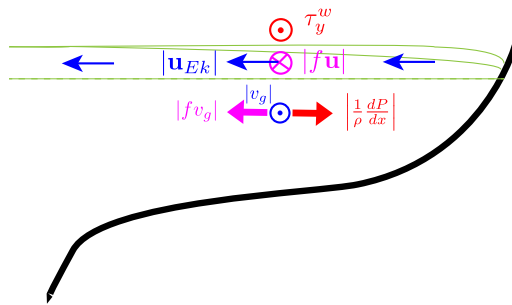


Figure 12.7: After the Ekman transport has been active for a while, the sea level drops at the coast (green surface), and an interior geostrophic balanced flow is set up.

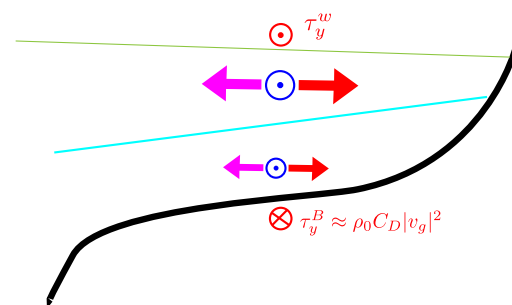
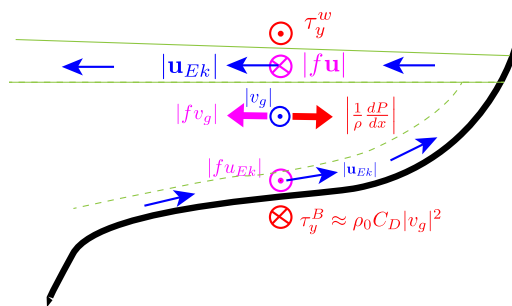


Figure 12.8: Sketch of pressure gradient changes if there are tilted isopycnals (cyan line) in the section, as observed in the coastal data. The tilted isopycnal counteracts the surface tilt, slowing the barotropic flow.

12.3 Exercises

Geostrophic balance two stations Calculate the geostrophic flow at two stations 50 km apart, assuming $f = 10^{-4} \text{ rad s}^{-1}$. Approximate the density measured at each location as 50-m blocks of homogeneous density:

z [m]	$\rho_A [\text{kg m}^{-3}]$	$\rho_B [\text{kg m}^{-3}]$
0–50	1021	1020
50–100	1022	1021
100–150	1025	1022
150–200	1026	1023

- If the surface pressure gradient is zero, what is the internal pressure gradient at 50, 100, 150, and 200 m? (do not drop too many decimal places!)
- What is the direction and strength of the *geostrophic* velocity at the 4 depths?
- If there is a surface pressure gradient, what must its direction and strength be so that the geostrophic flow is zero at 200 m?
- How large must the surface difference be at the two stations?
- What is the new flow at the other depths in the presence of this surface gradient?

Geostrophic flow Gulf Stream Consider the temperature section across the Gulf Stream (figure 12.9). Assuming a level of no motion at approximately 1500 m, what does

- the sea-surface tilt look like?
- the interior velocity look like (just a sketch)?
- where would you expect the Gulf Stream to be the fastest?

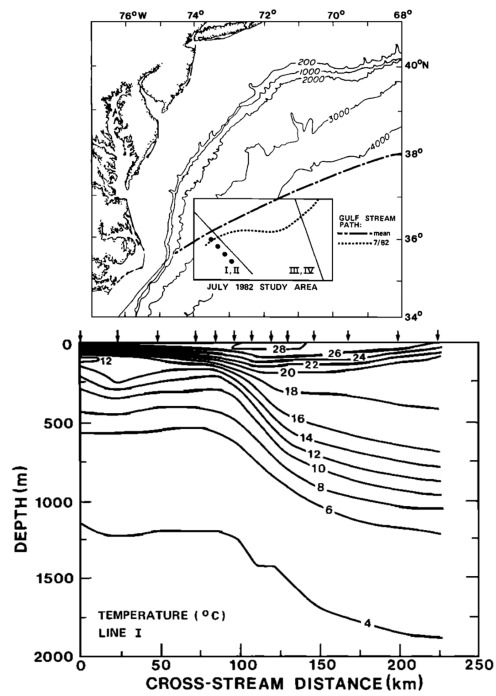


Figure 12.9: Location of measurements in the Gulf Stream, and plot of temperature across the section [?].